

## Equations (Senior UKMT)

These questions must be attempted without a calculator

Topics covered in the questions below may not necessarily be from the topic of the title.

1. The average of five consecutive integers is 10. What is the sum of the second and fourth of these integers?  
A 6                      B 10                      C 20                      D 21                      E can't be sure
  
2. Peri the winkle starts at the point (1, 1). Each day Peri crawls from point (x, y) to point (y, x + y), so that at the end of the first day Peri has reached (1, 2).  
Where is Peri at the end of the sixth day?  
A (6, 7)                      B (6, 12)                      C (13, 21)                      D (21, 34)                      E (144, 233)
  
3. The factorial of  $n$ , written  $n!$ , is defined by  $n! = 1 \times 2 \times 3 \times \dots \times (n-2) \times (n-1) \times n$   
e.g.  $6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ .  
What is the smallest positive integer which is *not* a factor of  $50!$  ?  
A 51                      B 52                      C 53                      D 54                      E 55
  
4. One of the oldest sporting events in the world is the Kiplingcotes Derby, a horse race which has been held in the East Yorkshire Wolds almost every year since 1519. Each rider pays a fee of £4.25 to enter the race. The first prize in the race is the sum of £50, but the second prize is the total of the entry fees minus an administration cost of 25p per rider. In 2000, 18 riders competed in the Kiplingcotes Derby.  
How much greater than the first prize was the second prize?  
A £22                      B £26.50                      C £46                      D £72                      E £76.50
  
5. Pascal, Newton, Galileo and Fermat all took the same test. The average score of all four candidates was 16; Pascal and Newton had an average of 16, Pascal and Fermat had an average of 13, while Newton and Fermat had an average of 18.  
What was Galileo's score?  
A 14                      B 15                      C 16                      D 17                      E 18
  
6. When forest trees are planted 1 metre apart in a particular repeating pattern, covering a large area of ground, the density of trees is about 10 000 per hectare.  
If, instead, the trees were planted 2 metres apart in the same pattern, approximately how many trees per hectare would there be?  
A 2 500                      B 5 000                      C 10 000                      D 20 000                      E 40 000

7. Heather and Rachel each has some pennies. Heather has more than Rachel. In fact, the number of pennies that Heather has is the square of the number that Rachel has. The total number of pennies they have between them makes a whole number of pounds.

What is the smallest this total could be?

- A £1                      B £6                      C £57                      D £99                      E £101

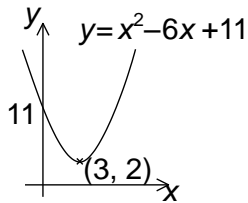
8. The ratio of Jon's age to Jan's age is 3 : 1. Three years ago the ratio was 4 : 1.

In how many years time will the ratio be 2 : 1?

- A 3                      B 6                      C 9                      D 12                      E 15

9. The curve  $y = x^2 - 6x + 11$  is rotated through  $180^\circ$  about the origin.

What is the equation of the new curve?



- A  $y = x^2 + 6x + 11$                       B  $y = x^2 - 6x - 11$                       C  $y = -x^2 + 6x - 11$   
D  $y = -x^2 - 6x + 11$                       E  $y = -x^2 - 6x - 11$

10. For how many integer values of  $n$  does the equation  $x^2 + nx - 16 = 0$  have integer solutions?

- A 2                      B 3                      C 4                      D 5                      E 6

11. A function  $f$  has the property that  $f(n + 3) = \frac{f(n) - 1}{f(n) + 1}$  for all positive integers  $n$ .

Given that  $f(2002)$  is non-zero, what is the value of  $f(2002) \times f(2008)$ ?

- A 1                      B -1                      C 2                      D -2                      E more information needed

12. A positive number  $a = [a] + \{a\}$  where  $[a]$  is the integer part of  $a$  and  $\{a\}$  is the fractional part of  $a$ .

Given that  $x + [y] + \{z\} = 4.2$ ,  $y + [z] + \{x\} = 3.6$ ,  $z + [x] + \{y\} = 2.0$  and  $x, y, z > 0$ , what is the value of  $\{y\}$ ?

- A 0.1                      B 0.3                      C 0.5                      D 0.7                      E 0.9