

Area (Senior UKMT)

These questions must be attempted without a calculator

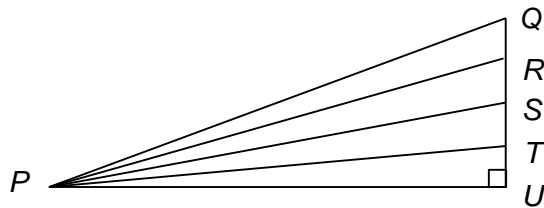
Topics covered in the questions below may not necessarily be from the topic of the title.

1. How many squares of area 1 cm^2 have a total perimeter length which is equal to the perimeter of a single square of area 4 cm^2 ?

A 2 B 4 C 8 D 16 E 32

2. Triangle PQU has a right angle at U . The points R , S and T divide the side QU into quarters.

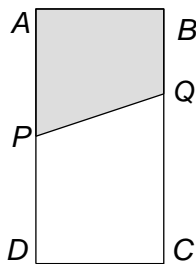
Which of the following statements about the areas of the triangles PQR , PRS , PST , PTU is true?



- A All have the same area B ΔPQR is biggest C ΔPRS is biggest
 D ΔPST is biggest E ΔPTU is biggest

3. $ABCD$ is a rectangle. P is the midpoint of AD ; the length of BQ is one third of the length of BC .

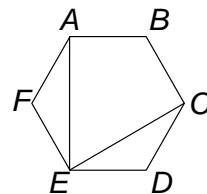
What fraction of the area of the rectangle is the area of the shaded quadrilateral $ABQP$?



- A $\frac{5}{12}$ B $\frac{2}{5}$ C $\frac{3}{8}$ D $\frac{1}{3}$ E $\frac{7}{16}$

4. $ABCDEF$ is a regular hexagon of area 60.

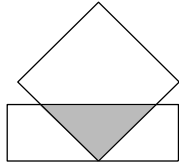
What is the area of the kite-shaped figure $ABCE$?



- A $20\sqrt{3}$ B 40 C 49 D 50 E 51

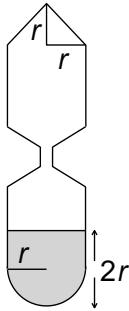
5. The diagram shows a 2×2 square and a 3×1 rectangle. One vertex of the square lies on a side of the rectangle. The sides of the rectangle are parallel to the diagonals of the square.

What is the area of the shaded triangle?

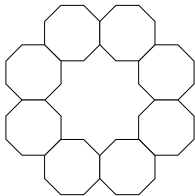


- A $\frac{1}{2}$ B 1 C $\frac{3}{2}$ D 2 E $\frac{5}{2}$
6. One end of an egg-timer is a hemisphere of radius r ; the other end is a cone of radius r and height r . Both ends are attached to cylinders of radius r . When the hemisphere is at the bottom, the sand in the egg-timer comes to a height $2r$ above the lowest point.

What is the corresponding height of the sand when the egg-timer has been turned over and all the sand has been allowed to run through to the other end?



- A $\frac{4}{3}r$ B $\frac{5}{3}r$ C $2r$ D $(\frac{1}{2} + \frac{\pi}{4})r$ E $\frac{7}{3}r$
7. Eight identical regular octagons are placed edge to edge in a ring in such a way that a symmetrical star shape is formed by the interior edges. If each octagon has sides of length 1, what is the area of the star?



- A $5+10\sqrt{2}$ B $8\sqrt{2}$ C $9+4\sqrt{2}$ D $16-4\sqrt{2}$ E $8+4\sqrt{2}$
8. It takes two weeks to clean the 3312 panes of glass in the 6000m^2 glass roof of the British Museum, a task performed once every two years.

Assuming that all the panes are equilateral triangles of the same size, roughly how long is the side of each pane?

- A 50 cm B 1 m C 2 m D 3 m E 4 m

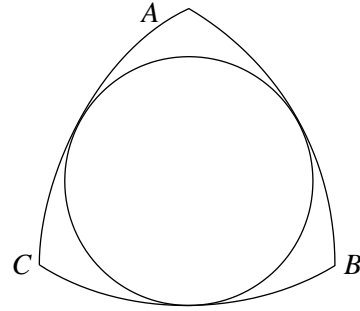
9. A triangle is cut from the corner of a rectangle. The resulting pentagon has sides of length 8, 10, 13, 15 and 20 units, though not necessarily in that order.

What is the area of the pentagon?

- A 252.5 B 260 C 270 D 275.5 E 282.5

10. The curvy shape ABC shown here is called a *Reuleaux triangle* (after the French engineer *Franz Reuleaux* (1829 - 1905)). Its perimeter consists of three equal arcs AB , BC , CA each with the same radius and centred at the opposite vertex. In the *Reuleaux triangle* shown, each arc has radius 3cm.

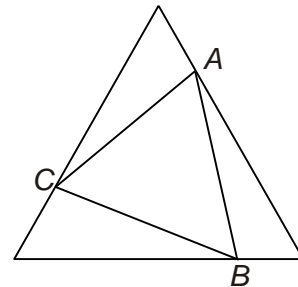
What is the area (in cm^2) of the inscribed circle?



- A $6\pi(2 - \sqrt{3})$ B $9\pi / 4$ C $2\pi(3 - \sqrt{3})$ D $3\pi / 4$ E 9π

11. The outer equilateral triangle has area 1. The points A , B , C are a quarter of the way along the sides as shown.

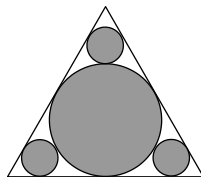
What is the area of the equilateral triangle ABC ?



- A $\frac{3}{8}$ B $\frac{7}{16}$ C $\frac{1}{2}$ D $\frac{9}{16}$ E $\frac{5}{8}$

12. The diagram shows four touching circles, each of which also touches the sides of an equilateral triangle with sides of length 3.

What is the area of the shaded region?



- A $\frac{11\pi}{12}$ B π C $\frac{(4 + \sqrt{3})\pi}{6}$ D $\frac{(3 + \sqrt{3})\pi}{4}$ E $\frac{37\pi}{36}$