Area and Volume Factors of Similar Shapes

Starter

1. A square of length 4 units is enlarged by a length factor of 3.
   (a) Find the area:
      (i) before and
      (ii) after the enlargement.
   (b) State the area factor from the small square to the big square.
   (c) What is the connection between the length factor of 3 and the area factor you calculated?

2. A cube of length 3 units is enlarged by a length factor of 2.
   (a) Find the volume:
      (i) before and
      (ii) after the enlargement.
   (b) State the volume factor from the small cube to the big cube.
   (c) What is the connection between the length factor of 2 and the volume factor you calculated?

Notes

From the starter, the following relationships can be seen:

$$\text{Area factor} = \text{Length factor}^2$$

$$\text{Volume factor} = \text{Length factor}^3$$

E.g. 1 Two triangles, P and Q, are similar. Triangle P has base length 3 cm and area 5 cm², while Q has base length 9 cm. Find the area of triangle Q.

Working: Length factor ***from P to Q*** $= \frac{9}{3} = 3$ ***small to big so Lf > 1***

$$\text{Area factor} = \text{Length factor}^2 = 3^2 = 9$$

$$\text{Area of triangle Q} = \text{Area factor} \times \text{Area of P}$$
$$= 9 \times 5 = 45 \text{ cm}^2$$

E.g. 2 The radii of the bases of two similar cones are 6 cm and 15 cm. Given that the smaller cone has volume 60π cm³, find the volume of the other cone. Give your answer in terms of π.

E.g. 3 Cylinders A and B are similar shapes and their radii are 3 cm and 2 cm respectively. Given that cylinder A has surface area 63π cm², find the surface area of B in terms of π.

Working: Length factor ***from A to B*** $= \frac{2}{3}$ **big to small so Lf < 1**

$$\text{Area factor} = \text{Length factor}^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\text{Surface area of cylinder B} = \text{Area factor} \times \text{Area of A}$$
$$= \frac{4}{9} \times 63\pi = 28\pi \text{ cm}^2$$

E.g. 4 Two triangles, A and B, are similar. Triangle A has area 20 cm^2 and triangle B has area 125 cm^2. Given that the base length of triangle A is 12 cm, calculate the base length of triangle B.

N.B. $\text{Length factor} = \sqrt{\text{Area factor}}$
$\text{Length factor} = \sqrt[3]{\text{Volume factor}}$

Ratios
Similarly if the length ratio between two similar shapes is $a : b$, then the

Area ratio	$a^2 : b^2$
Volume ratio	$a^3 : b^3$

E.g. 5 Squares A and B have side lengths in the ratio $2 : 3$. Square A has sides of length 8 cm.
(a) Find the side length of B.
(b) Find the ratio of the area of A to the area of B.

Working: (a) Length ratio is $2 : 3$
A has side 8 so the ratio is $8 : B$
From one ratio to the other multiply by 4, so side length of B is 12 cm

Video: Area factors
Video: Volume factors

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p454 E13.1 Qu 1-20 (odd)
A*-G class textbook:	p408 E13.6 Qu 1-20 (odd)
9-1 homework book:	p157 E13.1 Qu 1-8
A*-G homework book:	p115 E13.6 Qu 1-8

Summary
$\text{Area factor} = \text{Length factor}^2$
$\text{Volume factor} = \text{Length factor}^3$

Homework book answers (only available during a lockdown)