

Conditional Probability

Starter

1. **(Review of last lesson)** A bag has 5 red discs and 3 blue discs in it. Two discs are removed.
- Draw a tree diagram.
- Find the probability of choosing
- 1 red disc
 - at least 1 red disc
 - a blue disc given that you have already taken a red disc

Notes

$P(B \text{ given } R)$ means “the probability of B given that R has already occurred.”
i.e. R comes before B.

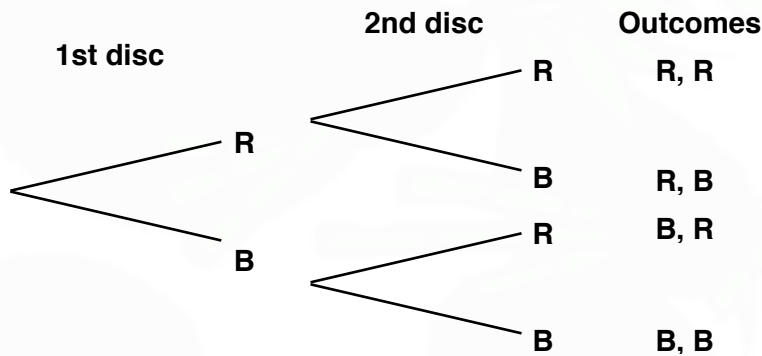
N.B. The word “*if*” could be used instead of “*given*”

Notation

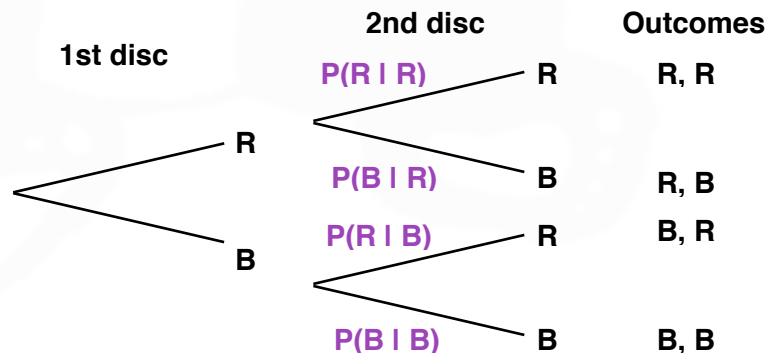
A vertical line “|” is used to denote given so $P(B \text{ given } R) = P(B | R)$

E.g. 1 Which branch is given by:

- (a) $P(R | R)$ (b) $P(B | B)$ (c) $P(R | B)$ (d) $P(B | R)$?



Working:



What is the connection between $P(R, B)$ and $P(B | R)$?

Multiply across the branches R and B: $P(R) \times P(B | R) = P(R, B)$

Rearranging gives: $P(B | R) = \frac{P(R, B)}{P(R)}$

In general,

$$P(B|A) = \frac{P(A, B)}{P(A)}$$

Special case for independent events

If events A and B are independent then $P(A, B) = P(A) \times P(B)$ so

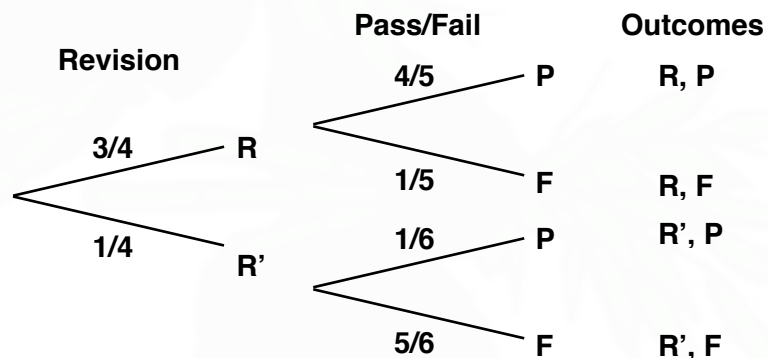
$$P(B|A) = \frac{P(A, B)}{P(A)} = \frac{P(A) \times P(B)}{P(A)} = P(B)$$

N.B. When deciding how to order your events, think about the chronological order that they happen i.e. what comes first.

E.g. 2 The probability that Tim passing a maths exam given that he revises is $\frac{4}{5}$ and $\frac{1}{6}$ if he does not revise. The probability that he revises is $\frac{3}{4}$.

- (a) Draw a tree diagram.
- (b) Find the probability that he passes the exam.

Working: (a) Revision comes before the exam so that is the first branch
Remember: the sum of the probabilities on the branches = 1



(b)
$$P(\text{Tim passes}) = P(R, P) + P(R', P)$$

$$= \frac{3}{4} \times \frac{4}{5} + \frac{1}{4} \times \frac{1}{6} = \frac{77}{120}$$

E.g. 3 In a town, 70% of children are vaccinated against flu. The probability of getting flu after having the vaccination is 0.1. The probability of getting flu given the person did not have the vaccination is 0.4.

- (a) Draw a tree diagram to represent this information.
- (b) Find the probability that a child chosen at random gets the flu.

N.B. “The probability of getting flu after having the vaccination is 0.1” \equiv
 “If you have the vaccination, the probability of getting flu is 0.1” \equiv
 “The probability of getting flu given a person had the vaccination is 0.1”

Hint: vaccination comes before the flu.

E.g. 4 The probability that a particular day is dry is $\frac{3}{10}$. The probability that Ripon City win on a dry day is $\frac{3}{8}$. Given that it is a wet day, the probability that they win is $\frac{3}{11}$.

- (a) Write down the value of $P(\text{not win} \mid \text{dry})$.
- (b) Calculate the probability they will win their next match.
- (c) Given that Ripon City won their match, what is the probability it was a dry day?

Hints: Although the question does not ask it, draw a tree diagram to help you.

Use the formula $P(B \mid A) = \frac{P(A, B)}{P(A)}$ for (c).

E.g. 5 A climatologist reports that the probability of rain on Friday is 0.21. If it rains on Friday, there is a 0.83 chance of rain on Saturday; if it doesn't rain on Friday, the chance of not raining on Saturday is 0.7.

- (a) Draw a tree diagram to represent this situation.
- (b) Use your diagram to calculate
 - (i) the probability of rain on Friday and Saturday
 - (ii) the probability of rain on only 1 day
 - (iii) the probability that it rains on Friday given it rains on only 1 day.

Video: [Conditional probability](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook:	p260 E8.2 Qu 1-9
A*-G class textbook:	p222 E8.2 Qu 1-9
9-1 homework book:	p90 E8.2 Qu 1-6
A*-G homework book:	p64 E8.2 Qu 1-6 (not 6b)

Summary

A vertical line “|” is used to denote “given” so $P(B \text{ given } R) = P(B \mid R)$

$$P(B \mid A) = \frac{P(A, B)}{P(A)}$$

[Homework book answers \(only available during a lockdown\)](#)