Congruent Triangles

Starter

1. **(Review of last lesson)** Jenny has 5 cards. The cards have a mean of 9 and a range of 6. Given that the numbers on the middle three cards in ascending order are 8, 9 and 10, find the numbers on the other two cards.

Congruent shapes are identical in size and shape.
Similar shapes are when one shape is the enlargement of another.

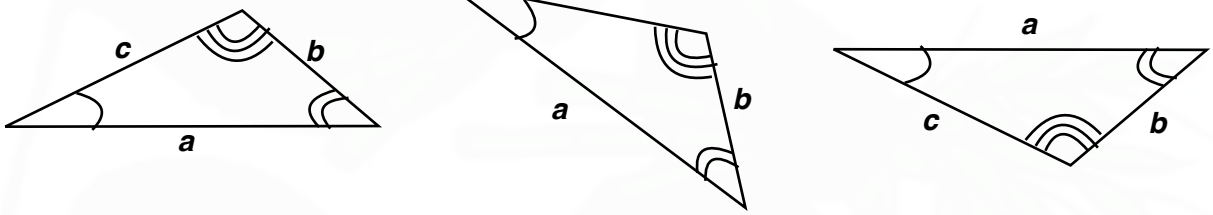
2. True or false: all rectangles are similar

3. All _____ are similar to each other. Name two shapes which could fit in the blank space.

Notes

Congruent triangles — two triangles are congruent when they are ***identical*** i.e. ***corresponding*** sides are equal and ***corresponding*** angles are equal.

The triangles below are all congruent. From the 1st triangle, the second one has been rotated and the 3rd one has been flipped but they are still the same triangle.



We can think of ***congruent*** as ***same shape***, ***same size***.

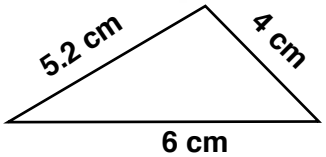
We do not need to know all sides and all angles to decide that ***two triangles*** are congruent.

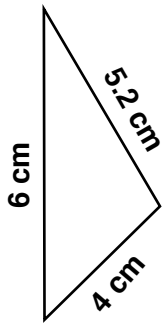
4 ways to check the congruency of triangles

The 4 ways to check whether 2 triangles are congruent are **SSS**, **SAS**, **ASA**, **RHS**.

1. **Side, Side, Side (SSS)**
All three sides of one triangle are equal to the corresponding on the other triangle.

This is known as Side, Side, Side or **SSS**



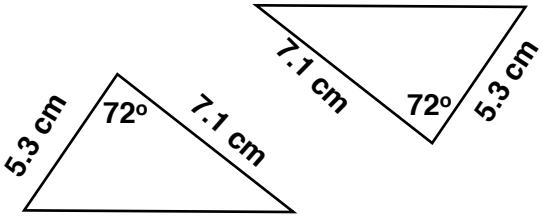


2. **Side, Angle, Side (SAS)**

Two sides and the ***angle between them*** are equal to the corresponding sides and angle of the other triangle.

This is known as Side, Angle, Side or **SAS**

N.B. The angle ***must be between*** the two sides



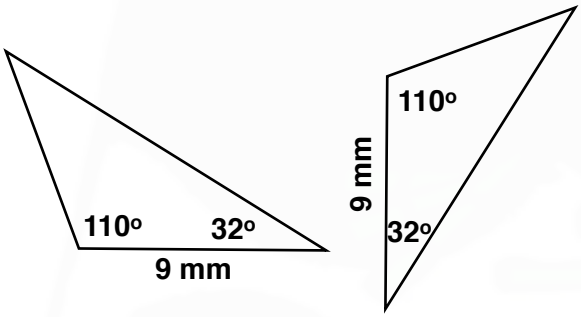
3. **Angle, Side, Angle (ASA)**

Two angles and ***any side*** are equal to the corresponding angles and side of the other triangles.

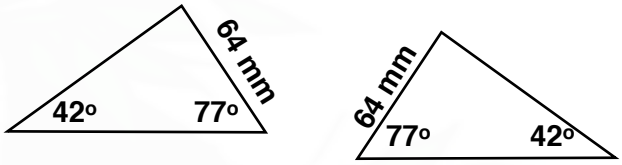
N.B. The side (S) does not need to be between the angles

This is known as Angle, Side, Angle or **ASA**

Example 1: the side is between the angles



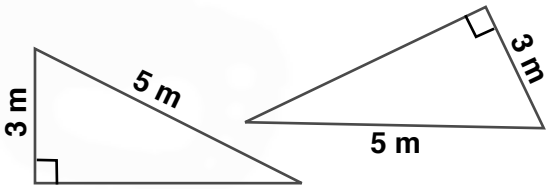
Example 2: the side is not between the angles



4. **Right-angle, Hypotenuse, Side (RHS)**

In two right-angled triangles, the ***hypotenuse*** and ***another side*** in one triangle are equal to the hypotenuse and the corresponding triangle of the other triangle.

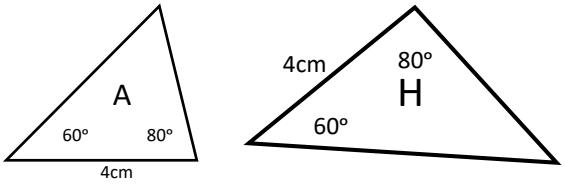
Given you have 2 sides of a right-angled triangle it is easy to see that you could work out the 3rd side and use SSS



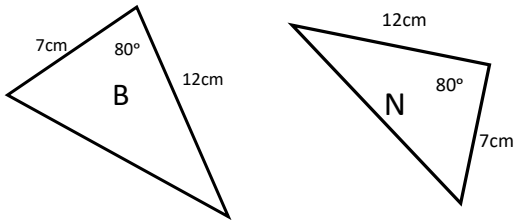
This is known as Right-angle, Hypotenuse, Side or **RHS**

E.g. 1 For the pairs of triangles state which of the 4 ways makes them congruent: SSS, SAS, ASA or RHS:

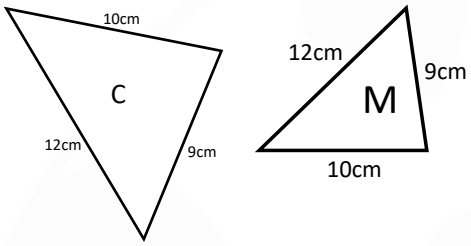
(a)



(b)



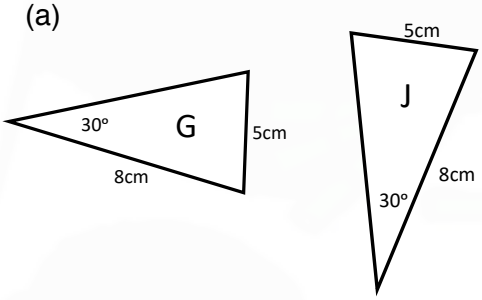
(c)



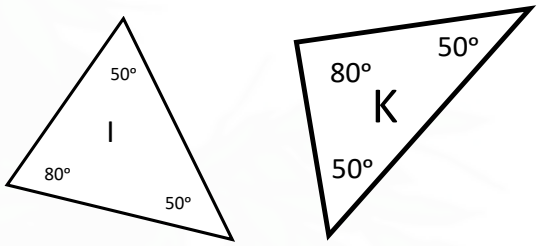
Working: (a) Two pairs of corresponding angles are equal and 1 corresponding pair of sides are equal so ASA

E.g. 2 Explain why these pairs of triangles are not necessarily congruent.

(a)

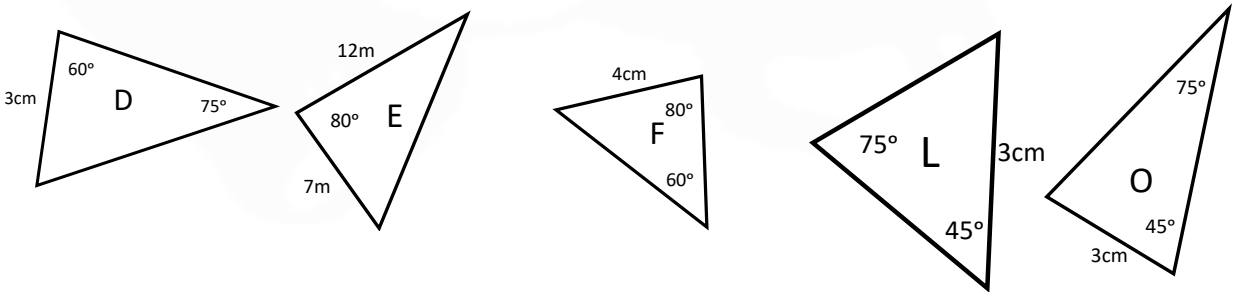


(b)



Working: (a) It looks like SAS but the angle is not between the two sides

E.g. 3 Find the congruent triangles from these triangles.
Hint: you may need to work out the 3rd angle



Working: L and O are congruent due to ASA — the 3 cm side is opposite to 75° and the 45° angle is next to the 3 cm side
Now consider triangle D. The 3rd angle is 45°. So again 3 cm is opposite 75° and the 45° angle is next to the 3cm side. So D is congruent to L and O.

N.B. If you have 2 angles, you can work out the third angle and this can help you decide if two triangles are congruent.

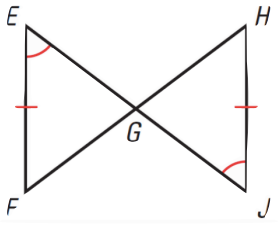
Angle notation
There are three notations you can use to denote an angle.

$$\hat{B} \equiv \widehat{ABC} \equiv \angle ABC$$

Proving two triangles are congruent
The examples above did not ask you to ***prove*** that the triangles were congruent. Proving congruency requires ***four statements***.

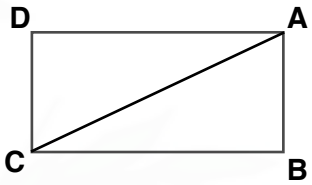
Note that all 4 ways to check congruency have 3 letters (SSS, SAS etc) so when proving whether two triangles are congruent, ***a statement is needed for each letter, which explains why the two sides or 2 angles are equal***. The 4th and final statement simply states which of the 4 ways has been used (e.g. Since we have SSS, the 2 triangles are congruent).

E.g. 4 Prove that triangle EFG is congruent to GHJ.



Working: Side: EF = HJ (given)
Angle: $\widehat{FEG} = \widehat{GJH}$ (given)
Angle: $\widehat{EGF} = \widehat{HGJ}$ (vertically opposite angles)
Since we have ASA, the triangles are congruent

E.g. 5 Prove that the triangle ABC is congruent to ACD in the rectangle.



Video: Congruent triangles
Video: Congruent and similar shapes

Solutions to Starter and E.g.s

Exercise
9-1 class textbook: p293 M9.7 Qu 1-9
A*-G class textbook: p256 E9.3 Qu 1-8
9-1 homework book: p100 M9.7 Qu 1-5, 8
A*-G homework book: p73 E9.3 Qu 1-5, 8
or Congruent Triangles Page 1: Qu 1, 2 ***and*** Page 2 Apply Qu 1-5

Summary
Congruent triangles — two triangles are congruent when they are ***identical*** i.e. ***corresponding*** sides are equal and ***corresponding*** angles are equal.
There are 4 ways to check whether 2 triangle are congruent:
SSS — ***all three sides*** of one triangle are equal to the corresponding on the other triangle.
SAS — ***two sides*** and the ***angle between them*** are equal to the corresponding sides and angle of the other triangle.
ASA — ***two angles*** and ***any side*** are equal to the corresponding angles and side of the other triangles.
N.B. The side (S) does not need to be between the angles
RHS — in two right-angled triangles, the ***hypotenuse*** and ***another side*** in one triangle are equal to the hypotenuse and the corresponding triangle of the other triangle.

Homework book answers (only available during a lockdown)
Answers Congruent Triangles ANSWERS