Connecting Area and Volume Factors

Starter
Last lesson we found that:

$$\text{Area factor} = \text{Length factor}^2$$
$$\text{Volume factor} = \text{Length factor}^3$$
$$\text{Length factor} = \sqrt{\text{Area factor}}$$
$$\text{Length factor} = \sqrt[3]{\text{Volume factor}}$$

N.B. When finding the volume factor from the area factor, or vice versa, always calculate the length factor as the first step

1. Given that the area factor is 25, find the:
   (a) length factor (b) the volume factor

2. Given that the volume factor is 27, find the area factor.

3. The volume ratio is $8 : 125$. Find the: (a) length ratio (b) area ratio.

Notes
N.B. We do not go straight from the area factor to the volume factor. Instead, we find the length factor first.

Setting out your work
When working with both area and volume factors in the same question, students find it useful to write the information is a table like the one below:

	Area	Volume
Small		
Big		

E.g. 1 The cross-section of a hexagonal prism has an area of 18cm^2. A similar prism has a cross-sectional area of 162 cm^2. If the volume of the first prism is 270 cm^3, what is the volume of the second prism?

Working: Enter the information we know in the table

	Area	Volume
Small	18	270
Big	162	x

To find x we go towards the unknown so ***small to big***
We need the volume factor, but ***first we calculate the length factor***
Area factor small to big $= \dfrac{162}{18} = 9$ **small to big so > 1**
Length factor small to big $= \sqrt{9} = 3$
Volume factor small to big $= 3^3 = 27$
Volume of second prism, $x = 27 \times 270 = 7290$ cm^3.

E.g. 2 A container has a surface area of 5000 cm^2 and a capacity of 12.8 litres. Find the surface area of a similar container which has a capacity of 5.4 litres.

E.g. 3 The masses of two similar objects are 24 kg and 81 kg respectively. If the surface area of the larger object is 540 cm^2, find the surface area of the smaller object.

Ratios

Length ratio: $a : b$
Area ratio: $a^2 : b^2$
Volume ratio: $a^3 : b^3$

E.g. 4 Two similar solids have surface areas in the ratio of $49 : 81$. Find the ratios of:
(a) their side lengths
(b) their volumes.

Working: (a) Area ratio is $49 : 81$
So length ratio is $\sqrt{49} : \sqrt{81}$
i.e. $7 : 9$

Video: Connecting area and volume factors

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p457 E13.2 Qu 1-12
A*-G class textbook:	p411 E13.7 Qu 1-7
9-1 homework book:	p158 E13.2 Qu 1-8
A*-G homework book:	p116 E13.7 Qu 1-7

Summary

Area factor $=$ Length factor^2
Volume factor $=$ Length factor^3
Length factor $= \sqrt{\text{Area factor}}$
Length factor $= \sqrt[3]{\text{Volume factor}}$
Setting out your work:

	Area	Volume
Small		
Big		

Length ratio: $a : b$
Area ratio: $a^2 : b^2$
Volume ratio: $a^3 : b^3$

Homework book answers (only available during a lockdown)