

Direct Proportion Equations (Linear)

Starter

1. (Review of Y9 material) If 6 pens cost £25, how much do 20 pens cost?
2. (Review of Y9 material) How far is 17 km in miles if $8 \text{ km} \approx 5 \text{ miles}$?

Notes

The questions from the starter involve quantities that are directly proportional to one another.

Two quantities are '**directly proportional**' if one quantity is a multiple of the other.

E.g. If one quantity is doubled, the other quantity is doubled. Link to straight line graphs.

E.g. If one quantity is trebled, the other quantity is trebled.

The **proportional** symbol is \propto .

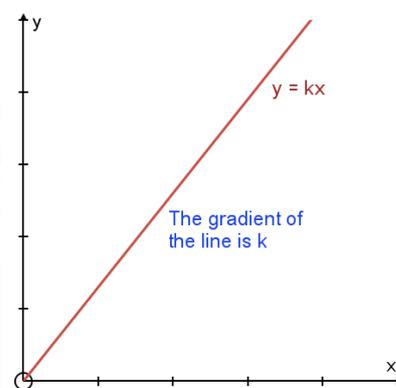
So if **y is directly proportional to x** , we write: $y \propto x$

The proportional symbol can be replaced by an "=" symbol when a **constant of proportionality**, k , is included.

$$y \propto x \quad \Rightarrow \quad y = kx$$

This is the equation of a straight line passing through the origin with gradient k .

Our aim is to use information, such as was given in the starter, to form an equation involving the quantities.



N.B. **Directly** proportional is a **linear** relationship

E.g. 1 The number of pens, n , is directly proportional to the cost, C in £.

- (a) Given that 8 pens cost £31, find an equation for C in terms of n pen.
- (b) How much would 165 pens cost? Give your answer to the nearest pence.

Working:

(a)	$C \propto n$	\Rightarrow	$C = kn$	(write formula with \propto and =)
	$C = 31, n = 8:$		$31 = 8k$	(substitute into the formula)
			$k = 3.875$	(solve for k)
			So $C = 3.875n$	(replace k by the value found)

(b)	When $n = 165,$	$C = 3.875 \times 165$	(substitute)
		$C = \text{£}639.38$	(nearest penny)

Success criteria – finding the proportion equation

1. Write an expression with the proportion symbol, \propto .
2. Replace the proportion symbol by " $= k$ ", where k is the **constant of proportionality**.
3. Use the given values to find the value of k .
4. Replace k in the equation from step 2 by the value found in step 3.

E.g. 2 A quantity P is directly proportional to Q . When $P = 235, Q = 5$.

- (a) Find the constant of proportionality.
- (b) Find P when $Q = 14$.
- (c) Find Q when P is 3901.

E.g. 3 The amount of money earned by Sasha, M , is directly proportional to the number of hours, h , she works. If she works for 9.5 hours she earns £155.80.

- (a) Express M in terms of h .
- (b) Using the equation formed in part (a), find out how many hours it would take her to earn £688.80.

Tables with direct proportion quantities

If you are asked to complete a table for directly proportional quantities, you will be given one complete set of data (e.g. in the table below $x = 7$ when $y = 42$).

With this information you can calculate the **constant of proportionality**, k , using the method above. Once you have the equation it is easy to complete the table.

E.g. 4 Complete the table given that x and y are directly proportional to one another.

x	7	3		-6	
y	42		84		-54

Working: Use the given values to find k :

$$\begin{aligned}y &\propto x &\Rightarrow & y = kx \\x = 7, y = 42: & & & 42 = 7k \\ & & & k = \dots\end{aligned}$$

The table can now be completed.

N.B. From the table it can be seen that $\frac{y}{x}$ is constant.

Video: [Direct proportion](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook: p143 M5.7 Qu 1-11; p145 E5.2 Qu 1, 6
A*-G class textbook: p134 E5.2 Qu 1-7
9-1 homework book: p50 M5.7 Qu 1-8
A*-G homework book: p37 E5.2 Qu 1-5

Summary

Two quantities are '**directly proportional**' if one quantity is a multiple of the other.

If y is **directly proportional to** x , we write: $y \propto x \Rightarrow y = kx$

Success criteria – finding the proportion equation

1. Write an expression with the proportion symbol, \propto .
2. Replace the proportion symbol by " $= k$ ", where k is the **constant of proportionality**.
3. Use the given values to find the value of k .
4. Replace k in the equation from step 2 by the value found in step 3.

[Homework book answers \(only available during a lockdown\)](#)