Product Rule for Counting

Starter

1. **(Review of last lesson)** A tetrahedron has the numbers 1, 3 , 4 and 4 written on its faces. A coin has the numbers 2 and 5 on its sides. The tetrahedron and coin and both tossed and the sum of the result is found.
   (a) Draw a probability space diagram to show the possible outcomes.
   (b) Find the probability that the sum is:
   (i) 6 (ii) 6 or 7 (iii) greater than 4.

2. A password has one entry. The password can be made up of letters (upper or lower case) and digits. How many different passwords are there?

3. If you have two entries, how many different passwords are there?

Notes

When solving such problems, draw a box for each entry and write the number of ways each box can be filled inside the box. Then find the product of the numbers in the box.

For question 3 from the starter, we would draw two boxes and write 62 in each box because there are 62 different ways to have each entry.

62	62

Two entries

If there are p ways of doing the 1st entry and q ways of doing the 2nd entry, then the total number of ways of doing the two tasks is $p \times q$.

N.B. This can be extended to any number of entries (see ***E.g. 1***)

E.g. 1 A code has 3 entries — the first is the letter b or g (depending on whether the person is a boy or girl, the second is an odd digit and the third is a lower case letter from the alphabet.

Working: Three entries so 3 boxes: ☐ ☐ ☐

1st entry (box): b or g so 2 ways | **2** | | |

2nd entry: odd digit (i.e. 1, 3, 5, 7, 9) so 5 ways | **2** | **5** | |

3rd entry (box): lower case letter so 26 ways | **2** | **5** | **26** |

Total number of ways = 2 × 5 × 26 = 260 ***product of numbers in boxes***

n entries, all having the same number of ways

If there are n entries and each entry has w number of ways, then the total number of ways is w^n.
See starter, question 2.

E.g. 2 A password has 6 entries. The first 2 entries must be a capital letters but a letter cannot be repeated. The next 2 entries must be digits, again without repeat. The last 2 letters can be lower case letters, capital letters or digits with repeats allowed. How many different passcode are there?

Working: 6 entries so 6 boxes

1st entry: capital letter so 26 ways
2nd entry: capital letter but no repeat so 25 ways
3rd entry: digit so 10
4th entry: digit but no repeat so 9
5th and 6th entries: lower case letters, capital letters or digits so 62

26	**25**	**10**	**9**	**62**	**62**

$26 \times 25 \times 10 \times 9 \times 62^2 = 224, 874, 000$

E.g. 3 A code has 3 entries. Each entry must be a digit 0-9 and the overall code must be odd. How many different codes are there if the first entry cannot be zero?

E.g. 4 Peter looks in his freezer and sees 12 microwave meals. Assuming he eats one microwave meal per day, Monday-Friday, how many possible combinations are there?

E.g. 5 A restaurant has a menu with 3 choices for a starter, 5 choices for the main dish and 4 choices for dessert.

(a) How many possible 3-course meals are there?

(b) Given that all customers must choose a main meal, how many 2-course meals are there?

Video: Product rule for counting

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p240 E8.1 Qu 1-15 odd
A*-G class textbook:	No exercise
9-1 homework book:	p82 E8.1 Qu 1-8
A*-G homework book:	No exercise

Summary

Two entries — p ways of doing the 1st entry and q ways of doing the 2nd entry, then the total number of ways of doing the two tasks is $p \times q$.

n entries, all having the same number of ways — n entries and each entry has w number of ways, then the total number of ways is w^n.

Homework book answers (only available during a lockdown)