

Quadratic Sequences

Starter

1. **(Review of last lesson)**

Find the formula for the n th term of the linear sequence given by $u_1 = 8$, $u_{n+1} = u_n + 3$.

2. **(Review of previous material)**

Find the second differences for the sequence 4, 7, 12, 19, 28.

3. Consider the expression $u_n = an^2 + bn + c$. Find expressions in terms of a , b and c for:

(a) u_1

(b) u_2

4. **(Review of previous material)** Solve these simultaneous equations:

(a) $b + c = 7$

(b) $b + c = 8$

$2b + c = 12$

$2b + c = 16$

Notes

A **quadratic sequence** is of the form $u_n = an^2 + bn + c$ (quadratic because it includes a term in n^2).

A quadratic sequence is one whose **2nd differences are equal**.

Given that $u_n = an^2 + bn + c$, from the start we found that:

$a + b + c = \text{1st term}$

$4a + 2b + c = \text{2nd term}$

The value of a is **half the second difference**.

Success Criteria – finding the formula for a quadratic sequence $u_n = an^2 + bn + c$

1. Find the second differences
2. Halve the value of the second difference to get a
3. Form 2 equations:

$$a + b + c = \text{1st term}$$

$$4a + 2b + c = \text{2nd term}$$

4. Replace a and the 1st and 2nd terms in the equations and rearrange to form simultaneous equations
5. Solve the simultaneous equations to find b and c .

E.g. 1 Find an expression for the n th term of the quadratic sequence 4, 7, 12, 19, 28.

Working: From the starter the second differences are 2 $\Rightarrow a = \frac{1}{2} \times 2 = 1$

$$\begin{array}{l} a + b + c = \text{1st term:} \quad a + b + c = 4 \\ 4a + 2b + c = \text{2nd term:} \quad 4a + 2b + c = 7 \end{array}$$

Replace a by 1:

$$\begin{array}{l} 1 + b + c = 4 \\ 4 + 2b + c = 7 \end{array}$$

Rearranging:

$$\begin{array}{r} b + c = 3 \\ 2b + c = 3 \\ \text{Subtract} \quad -b \quad = 0 \\ \hline b = 0 \\ \text{Subst.} \quad 0 + c = 3 \\ \hline c = 3 \end{array}$$

$$\begin{array}{l} a = 1, b = 0, c = 3 \\ \text{So } u_n = 1n^2 + 0n + 3 = n^2 + 3. \end{array}$$

E.g. 2 Find an expression in the form $u_n = an^2 + bn + c$ for the sequence 0, 5, 12, 21, 32.

E.g. 3 Find an expression for the n th term of the quadratic sequence 6, 13, 24, 39, 58.

E.g. 4 Find an expression for the n th term of the quadratic sequence 5, 8, 15, 26, 41.

- Video:** [Quadratic sequences \(method 1\)](#)
- Video:** [Quadratic sequences \(method 2\)](#)
- Video:** [Quadratic sequences \(method 3\)](#)

[Solutions to Starter and E.g.s](#)

Exercise

- 9-1 class textbook: p394 E12.2 Qu 1-11
- A*-G class textbook: p355 E12.1 Qu 1-12
- 9-1 homework book: p133 E12.2 Qu 1-5
- A*-G homework book: p100 E12.1 Qu 1-5

Summary

A **quadratic sequence** is of the form $u_n = an^2 + bn + c$.
The value of a is **half the second difference**.

Finding the formula for a quadratic sequence $u_n = an^2 + bn + c$

1. Find the second differences
2. Halve the value of the second difference to get a
3. Form 2 equations:
$$\begin{array}{l} a + b + c = \text{1st term} \\ 4a + 2b + c = \text{2nd term} \end{array}$$
4. Replace a and the 1st and 2nd terms in the equations and rearrange to form simultaneous equations
5. Solve the simultaneous equations to find b and c .

[Homework book answers \(only available during a lockdown\)](#)