Special Sequences (including Geometric and Fibonacci)

Starter

1. **(Review of last lesson)** Find the next two terms in the sequence 5, 56, 137, 254, 413.

Notes

Here are the common types of sequence you will come across.

Arithmetic progression — add the same number each time (also called ***linear sequences***)

N.B. The number could be positive of negative.
The number we add is called the ***common difference*** and is given the letter d

Examples 4, 7, 10, 13,… add 3 each time
56, 49, 42, 35,… add −7 each time (or subtract 7 each time)

The common difference is found by:

$$d = \text{2nd term} - \text{1st term} = \text{3rd term} - \text{2nd term} = \ldots$$

Geometric progression — multiply by the same number each time.

N.B. The number could be a fraction and/or a negative.
The number we multiply by is called the ***common ratio*** and is given the letter r

Examples 3, 12, 48, 192,… multiply by 4 each time so $r = 4$
128, 64, 32, 16,… multiply by $\frac{1}{2}$ each time so $r = \frac{1}{2}$

The common ratio is found by:

$$r = \frac{\text{2nd term}}{\text{1st term}} = \frac{\text{3rd term}}{\text{2nd term}} = \ldots$$

Fibonacci sequence — the next term is the sum of the previous two terms

E.g.s 1, 1, 2, 3, 5, 8, 13,… this is the original Fibonacci sequence
3, 3, 6, 9, 15, 24,… any two terms could start a Fibonacci-like sequence

Other sequences

Square numbers 1, 4, 9, 16, 25, …

Cube numbers 1, 8, 27, 64, 125, 216, …

Triangular numbers 1, 3, 6, 10, 15, 21, 28, …

E.g. 1 Decide if these sequences are geometric sequences. If it is, write down the common ratio.
(a) 7, 21, 63, 189 (b) 10, 20, 30, 40

Working: (a)

$$\frac{\text{2nd term}}{\text{1st term}} = \frac{21}{7} = 3$$
$$\frac{\text{3rd term}}{\text{2nd term}} = \frac{63}{21} = 3$$
$$\frac{\text{4th term}}{\text{3rd term}} = \frac{189}{63} = 3$$

Since the values are all the same, the sequence is a geometric sequence.
The common ratio, r, is 3.

E.g. 2 Find the 6th term of the sequence 2, 10, 50, 250.

Working: $\frac{10}{2} = \frac{50}{10} = \frac{250}{50} = 5$ so the sequence is geometric

5th term $= 250 \times 5 = 1250$

6th term $= 1250 \times 5 = 6250$

E.g. 3 Write down the next 5 terms of these ***Fibonacci*** sequences: (a) 2, 5 (b) a, b

Working: (a)

3rd term $= 2 + 5 = 7$

4th term $= 5 + 7 = 12$

5th term $= 7 + 12 = 19$

6th term $= 12 + 19 = 31$

7th term $= 19 + 31 = 50$

Video: Fibonacci sequences

Video: Geometric progressions

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p386 M12.7 Qu 1-14
A*-G class textbook:	No exercise
9-1 homework book:	p130 M12.7 Qu 1-12
A*-G homework book:	No exercise

Summary

Arithmetic progression — add the same number each time (also called linear sequences)

The common difference, d: d = 2nd term – 1st term = 3rd term – 2nd term = …

Geometric progression — multiply by the same number each time.

The common ratio, r: $r = \frac{\text{2nd term}}{\text{1st term}} = \frac{\text{3rd term}}{\text{2nd term}} = \ldots$

Fibonacci sequence — the next term is the sum of the previous two terms

Square numbers 1, 4, 9, 16, 25, …

Cube numbers 1, 8, 27, 64, 125, 216, …

Triangular numbers 1, 3, 6, 10, 15, 21, 28, …

Homework book answers (only available during a lockdown)