Subscript Notation for Sequences (H)

Starter

1. **(Review of last lesson)**
   Find a formula for the nth term for these linear sequences:
   (a) 7, 16, 25, 34 (b) -5, -12, -19, -26

2. **(Review of last lesson)**
   Find the 11th term for the sequence whose nth term is given by $17n + 5$

Notes

We are used to sequences written using a function of n (e.g. nth term $= 4n - 3$) where the 5th term is found by replacing n by 5 in the formula

i.e. 5th term $= 4 \times 5 - 3 = 17$

In this lesson we will look at defining a sequence using an ***iterative formula***.

Subscript notation

Rather than writing "5th term" we can use ***subscript notation*** to make it quicker.

5th term $= u_5$ the $_5$ is called the ***subscript***

23th term $= u_{23}$

nth term $= u_n$

N.B. If the current term is u_7, the next term is u_8

If the current term is u_n, the next term is u_{n+1}

Defining sequences using an iterative formula

Sequences can be defined by an ***iterative formula***, where the next term is found from the previous one. ***Iterative formulae*** use ***subscript notation***.

An iterative formula is comprised of two parts:

1. The first term, u_1
2. A formula of how to find the next term from the previous one
   i.e. u_{n+1} is written in terms of u_n

For example: $u_1 = 4$,

$u_{n+1} = u_n + 3$ i.e. we find the next term by adding 3 to the previous one

E.g. 1 Find the first 4 terms in the sequence defined by $u_1 = 4$, $u_{n+1} = u_n + 3$

Working:

$u_1 = 4$

$u_2 = u_1 + 3 = 4 + 3 = 7$

$u_3 = u_2 + 3 = 7 + 3 = 10$

$u_4 = u_3 + 3 = 10 + 3 = 13$

The first 4 terms are 4, 7, 10, 13

N.B. After finding the first few terms and seeing that it is a linear sequence, we could calculate a formula for the nth term:

Term-to-term rule: $7 - 4 = 3 \Rightarrow 3n$

Term before the first: $4 - 3 = 1$

$\therefore$ nth term, $u_n = 3n + 1$

The problem with iterative formulae
To get the 50th term, you must first find the 2nd, 3rd,…, 49th terms so it is not an efficient method.

E.g. 2 Write down the first four terms of the sequences with the following definitions:
(a) $u_1 = 2, u_{n+1} = u_n + 9$
(b) $u_1 = 13, u_{n+1} = u_n - 5$
(c) $u_1 = 5, u_{n+1} = 2u_n + 3$

Going backwards with an iterative formula
When going backwards, write out an equation and solve it to find the previous term in the sequence.

E.g. 3 Let $u_{n+1} = 3u_n + 1$. If $u_3 = 16$, find u_1.

Working: We first need to find u_2 so we write the iterative formula for u_2 and u_3
$u_3 = 3u_2 + 1$
Replace u_3 by 16: $16 = 3u_2 + 1$
Solve: $15 = 3u_2$
$u_2 = 5$
Repeat the process to find u_1:
Replace u_2 by 5: $5 = 3u_1 + 1$
Solve: $4 = 3u_1$
$u_1 = \frac{4}{3}$

E.g. 4 Let $u_{n+1} = u_n + 7$. If $u_4 = 8$, find: (a) u_8 (b) u_1

E.g. 5 Let $w_{n+1} = 2w_n + 5$. If $w_3 = 31$, find: (a) w_6 (b) w_1

Video: Iteration

Solutions to Starter and E.g.s

Exercise
9-1 class textbook: p392 E12.1 Qu 1-8
A*-G class textbook: No exercise
9-1 homework book: p133 E12.1 Qu 1-10
A*-G homework book: No exercise

Summary
Subscript notation: nth term $= u_n$

Defining sequences using an iterative formula:
1. The first term, u_1
2. A formula of how to find the next term from the previous one
   i.e. u_{n+1} is written in terms of u_n

Homework book answers (only available during a lockdown)