Trial and Improvement

Starter

1. **(Review of last lesson)**
   I think of a number, square it and then add five times the original number. The result is 24.

Notes

The equation $x^3 + 5x = 175$ is more difficult to solve because it is not a quadratic. Instead we use ***trial and improvement*** to get to an answer to a specific accuracy, e.g. 2 decimal places.

Trial and improvement means that we substitute one value of x into the formula and based on what comes out, we choose a closer approximation to the answer to substitute in next. We do this repeatedly until we get an answer to the required degree of accuracy.

Change of sign method

The first step is to locate the solution between 2 consecutive integers and for that we use the ***change of sign method***. The clue is in the same: integer $x-$values are repeatedly substituted into the equation until one answer is positive and the next one is negative.

Success criteria — change of sign method

1. Rearrange the equation so that $f(x) = 0$
2. Substitute values into $f(x)$ until ***either*** $f(a) > 0$ and $f(a+1) < 0$ ***or*** $f(a) < 0$ and $f(a+1) > 0$ where a is an integer

N.B. The actual value of $f(a)$ is not needed, just whether it < 0 or > 0

An example will show you that it is not that difficult.

E.g. 1 Locate the root of the equation $x^3 + 5x = 175$ between two consecutive integers.

Working: Rearrange the equation so that $f(x) = 0$: $x^3 + 5x - 175 = 0$

So $f(x) = x^3 + 5x - 175$

$f(2) = 2^3 + 5 \times 2 - 175 < 0$ ***actual values are not required***

N.B. $f(2)$ means replace x by 2 in the expression

$f(4) = 4^3 + 5 \times 5 - 175 < 0$

$f(5) = 5^3 + 5 \times 5 - 175 < 0$

$f(6) = 6^3 + 5 \times 6 - 175 > 0$ ***change of sign***

So the solution of $x^3 + 5x - 175 = 0$ lies between 5 and 6

E.g. 2 Locate the root of the equation $x^3 = 51 - 2x$ between two consecutive integers.

Trial and improvement

Let's do an example so you can see the method in action.

E.g. 3 Solve the equation $x^3 + 5x = 175$ to 1 d.p.

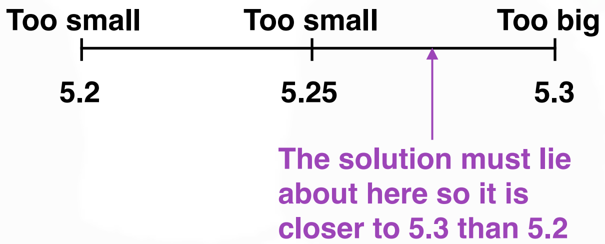
Working: Rearrange the equation so that $f(x) = 0$: $x^3 + 5x - 175 = 0$
So $f(x) = x^3 + 5x - 175$
From ***E.g. 1***, we know that $f(5) < 0$ (so $x = 5$ is too small) and $f(6) > 0$ (so $x = 6$ is too big)
N.B. Too big (or too small) means that the $x-$value is too big (or too small)

$x-$value	$f(x)$	Too big/small
5		Too small
6		Too big
5.5	$5.5^3 + 5 \times 5.5 - 175 = 18.875$	Too big
5.3	$5.3^3 + 5 \times 5.3 - 175 = 0.377$	Too big
5.2	$5.2^3 + 5 \times 5.2 - 175 = -8.392$	Too small

At this point we that the root is between 5.2 and 5.3.
It looks like it is closer to 5.3 than to 5.2 but must do one more calculation to confirm this.
Our final calculation must be ***half-way between the consecutive values*** to required accuracy — in this case half-way between 5.2 and 5.3.

$5.25^3 + 5 \times 5.25 - 175 = -4.04...$ i.e. too small

This diagram can help you to decide whether it is 5.2 or 5.3:



The solution to $x^3 + 5x = 175$ is 5.3 to 1 d.p.

Success Criteria — trial and improvement
If you have carried out the change of sign method, you will already have the solution between two consecutive integers and can enter these in the table straight-away.

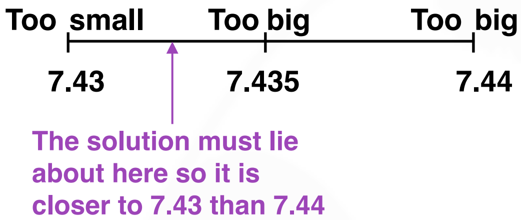
1. Draw a table with 4 columns: $x-$value, $f(x)$, Too big/small

$x-$value	$f(x)$	Too big/small

2. Choose $x-$values until consecutive integers give totals that are too small and too big.

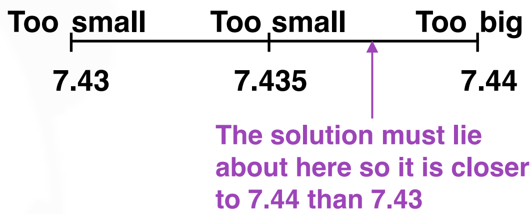
3. When changing accuracy of the $x-$value, (i.e. from integers to 1 d.p.), always choose the middle value.
4. Continue to improve the solution until you have it between two consecutive values to the required accuracy.
   E.g. if required to 2 d.p. you have say 7.43 being too big and 7.44 being too big
5. The final calculation is half-way between the consecutive values to the required accuracy
   E.g. for the above the final calculation would be substituting 7.435
6. Use the following diagrams below to help.

$x = 7.435$ **is too big**



Solution is $x = 7.43$ to 2 d.p.

$x = 7.435$ **is too small**



Solution is $x = 7.44$ to 2 d.p.

E.g. 4 Solve the equation $x^3 = 51 - 2x$ to 1 d.p.

E.g. 5 A solution to $x^3 - 3x = 170$ lies between 5 and 6. Find the solution correct to 1 d.p.

E.g. 6 A solution to $2x^3 + 7x = 100$ lies between 3.3 and 3.4. Find the solution correct to **2 d.p.**

Video: Trial and improvement

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p179 E6.4 Qu 1-9
A*-G class textbook:	p155 M6.5 Qu 1-12
9-1 homework book:	p64 E6.4 Qu 1-4
A*-G homework book:	p43 M6.5 Qu 1-4

Summary
Change of sign method

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N.B. The actual value of $f(a)$ is not needed, just whether it < 0 or > 0

Trial and improvement

If you have carried out the change of sign method, you will already have the solution between two consecutive integers and can enter these in the table straight-away.

1. Draw a table with 4 columns: $x-$value, $f(x)$, Too big/small

$x-$**value**	$f(x)$	**Too big/small**

2. Choose $x-$values until consecutive integers give totals that are too small and too big.
3. When changing accuracy of the $x-$value, (i.e. from integers to 1 d.p.), always choose the middle value.
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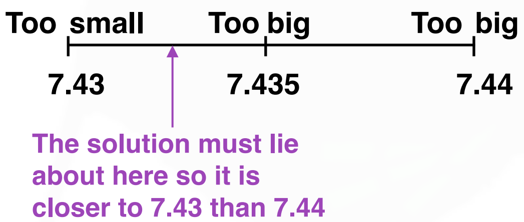
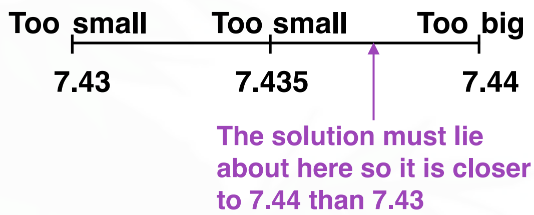


Solution is $x = 7.43$ to 2 d.p.

$x = 7.435$ **is too small**



Solution is $x = 7.44$ to 2 d.p.

Homework book answers (only available during a lockdown)