

Volume of Spheres, Pyramids and Cones

Starter

1. (Review of last lesson) The surface area of a cylinder is 80π cm². Given that the height of the cylinder is 3 cm, calculate its radius.

Notes

Volume of a sphere

The formula for the volume of a sphere is:

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3 \quad \text{where } r \text{ is the radius}$$

E.g. 1 Find the volume of sphere whose radius is 5 cm.

Volumes of pyramids and cones

The volume of a pyramid is given by:

$$\text{Volume of a pyramid} = \frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$$

For a cone, since its base is a circle we get the formula:

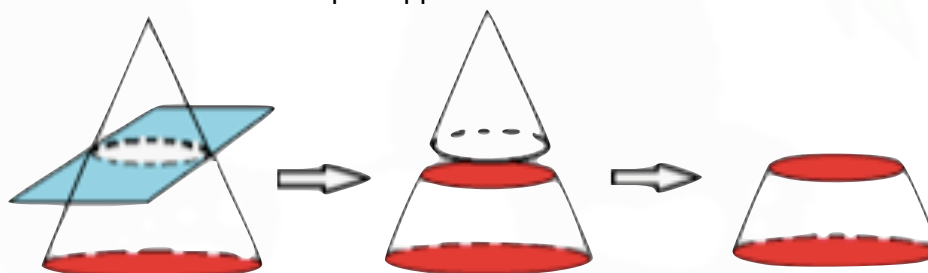
$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h \quad \text{where } r = \text{base radius}$$
$$h = \text{perpendicular height}$$



E.g. 2 Find the exact volume of a cone whose base radius is 5 cm and whose perpendicular height is 12 cm. Give your answer in terms of π .

Volume of a frustum

A frustum is a cone which has had the top chopped off.



$$\text{Volume of a frustum} = \text{Volume of original cone} - \text{Volume of removed cone}$$

N.B. For a frustum, the radii and perpendicular heights of the removed cone and the original cone are in the same ratio. Therefore, the original cone and the cone that was removed are **similar** solids.

Remember: Volume factor = Length factor³

E.g. 3 A cone of height 6 cm is removed the top of a larger cone to form a frustum. The radius of the top of the frustum is 4 cm and the radius of the base of the frustum is 12 cm. Calculate the volume of the frustum.

Working: The radii of the bases of the removed cone and original cone are 4 and 12

$$\text{So the length factor} = \frac{12}{4} = 3$$

The original cone is three times taller than the removed cone

$$\text{So height of original cone} = 3 \times 6 = 18$$

Volume of frustum = Volume of original cone – Volume of removed cone

$$\begin{aligned} &= \frac{1}{3}\pi \times 12^2 \times 18 & - & \frac{1}{3}\pi \times 4^2 \times 6 \\ &= 864\pi & - & 32\pi \\ &= 832\pi \end{aligned}$$

Alternatively: Length factor = $\frac{12}{4} = 3$

$$\text{Volume factor} = 3^3 = 27$$

Volume of original cone = 27 × volume of removed cone

$$\begin{aligned} &= 27 \times \frac{1}{3}\pi \times 4^2 \times 6 \\ &= 27 \times 32\pi \\ &= 864\pi \end{aligned}$$

Volume of frustum = Volume of original cone – Volume of removed cone

$$\begin{aligned} &= 864\pi - 32\pi \\ &= 832\pi \end{aligned}$$

Video: [Volume of a sphere](#)

Video: [Volume of a cone](#)

Video: [Volume of a frustum](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook: p441 M13.7 Qu 1-18

A*-G class textbook: p396 E13.4 Qu 1-18

9-1 homework book: p152 M13.7 Qu 1-7

A*-G homework book: p111 E13.4 Qu 1-7

Summary

Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius

Volume of a pyramid = $\frac{1}{3}$ × area of base × perpendicular height

Volume of a cone = $\frac{1}{3}\pi r^2 h$ where r = base radius

h = perpendicular height

Volume of a frustum = Volume of original cone – Volume of removed cone

Volume factor = Length factor³

[Homework book answers \(only available during a lockdown\)](#)