

Area and Volume Factors of Similar Shapes

Starter

1. A square of length 4 units is enlarged by a length factor of 3.
- Find the area:
 - before and
 - after the enlargement.
 - State the area factor from the small square to the big square.
 - What is the connection between the length factor of 3 and the area factor you calculated?

Working:

- $4^2 = 16$
 - $12^2 = 144$
- Area factor from small to big = $\frac{144}{16} = 9$
- $3^2 = 9$

2. A cube of length 3 units is enlarged by a length factor of 2.
- Find the volume:
 - before and
 - after the enlargement.
 - State the volume factor from the small cube to the big cube.
 - What is the connection between the length factor of 2 and the volume factor you calculated?

Working:

- $3^3 = 27$
 - $6^3 = 216$
- Volume factor from small to big = $\frac{216}{27} = 8$
- $2^3 = 8$

- E.g. 1** Two triangles, P and Q, are similar. Triangle P has base length 3 cm and area 5 cm², while Q has base length 9 cm. Find the area of triangle Q.

Working: Length factor *from P to Q* = $\frac{9}{3} = 3$ *small to big so Lf > 1*

Area factor = Length factor² = $3^2 = 9$

Area of triangle Q = Area factor × Area of P

= $9 \times 5 = 45 \text{ cm}^2$

- E.g. 2** The radii of the bases of two similar cones are 6 cm and 15 cm. Given that the smaller cone has volume $60\pi \text{ cm}^3$, find the volume of the other cone. Give your answer in terms of π .

Working: Length factor *from small to big* = $\frac{15}{6} = \frac{5}{2}$ *small to big so Lf > 1*

Volume factor = Length factor³ = $\left(\frac{5}{2}\right)^3 = \frac{125}{8}$

Volume of big cone = volume factor × volume of small cone

= $\frac{125}{8} \times 60\pi = \frac{1875}{2}\pi = 937.5\pi \text{ cm}^3$

E.g. 3 Cylinders A and B are similar shapes and their radii are 3 cm and 2 cm respectively. Given that cylinder A has surface area 63π cm², find the surface area of B in terms of π .

Working: Length factor *from A to B* = $\frac{2}{3}$ **big to small so Lf < 1**

$$\text{Area factor} = \text{Length factor}^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$
$$\begin{aligned}\text{Surface area of cylinder B} &= \text{Area factor} \times \text{Area of A} \\ &= \frac{4}{9} \times 63\pi = 28\pi \text{ cm}^2\end{aligned}$$

E.g. 4 Two triangles, A and B, are similar. Triangle A has area 20 cm² and triangle B has area 125 cm². Given that the base length of triangle A is 12 cm, calculate the base length of triangle B.

Working: Area factor *from A to B* = $\frac{125}{20} = \frac{25}{4}$ **from small to big so Af > 1**

$$\text{Length factor} = \sqrt{\text{Area factor}} = \sqrt{\frac{25}{4}} = \frac{5}{2}$$
$$\begin{aligned}\text{Base length of triangle B} &= \text{Length factor} \times \text{Base of A} \\ &= \frac{5}{2} \times 12 = 30 \text{ cm}\end{aligned}$$

E.g. 5 Squares A and B have side lengths in the ratio 2 : 3. Square A has sides of length 8 cm.
(a) Find the side length of B.
(b) Find the ratio of the area of A to the area of B.

Working: (a) Length ratio is 2 : 3
A has side 8 so the ratio is 8 : B
From one ratio to the other multiply by 4, so side length of B is 12 cm

(b) Area ratio is 2² : 3²
i.e. 4 : 9

Video: [Area factors](#)
Video: [Volume factors](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook: p454 E13.1 Qu 1-20 (odd)
A*-G class textbook: p408 E13.6 Qu 1-20 (odd)
9-1 homework book: p157 E13.1 Qu 1-8
A*-G homework book: p115 E13.6 Qu 1-8

[Homework book answers \(only available during a lockdown\)](#)