

## Congruent Triangles

### Starter

1. **(Review of last lesson)** Jenny has 5 cards. The cards have a mean of 9 and a range of 6. Given that the numbers on the middle three cards in ascending order are 8, 9 and 10, find the numbers on the other two cards.

**Working:** Let the other 2 cards be  $x$  and  $y$  so  $x, 8, 9, 10, y$  are the cards.  
 The cards have a mean of 9: 
$$\frac{x + 8 + 9 + 10 + y}{5} = 9$$
  

$$x + y + 27 = 45$$
  

$$x + y = 18$$
  
 The cards have a range of 6: 
$$y - x = 6$$
  

$$-x + y = 6$$
  
 Adding the equations eliminates  $x$ : 
$$2y = 24 \quad \text{so} \quad y = 12$$
  
 Substituting: 
$$x + 12 = 18 \quad \text{so} \quad x = 6$$
  
 The other 2 cards are 6 and 12.

**Congruent** shapes are identical in size and shape.

**Similar** shapes are when one shape is the enlargement of another.

2. True or false: all rectangles are similar

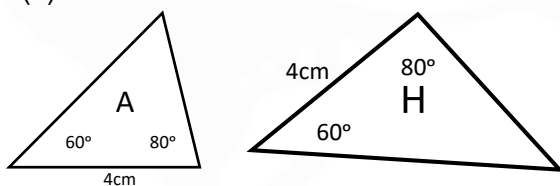
**Working:** False, a 15 cm by 5 cm rectangle is not an enlargement of a 5 cm by 2 cm rectangle

3. All \_\_\_\_\_ are similar to each other. Name two shapes which could fit in the blank space.

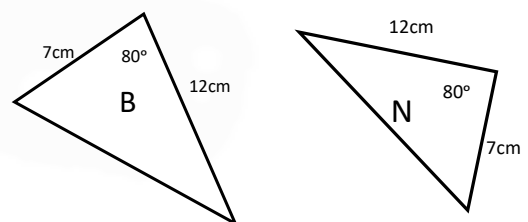
**Working:** Circles, squares (equilateral triangles or any regular polygon are also possible)

**E.g. 1** For the pairs of triangles state which of the 4 ways makes them congruent: SSS, SAS, ASA or RHS:

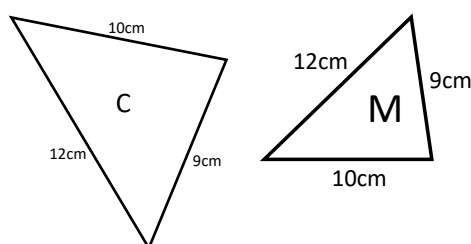
(a)



(b)

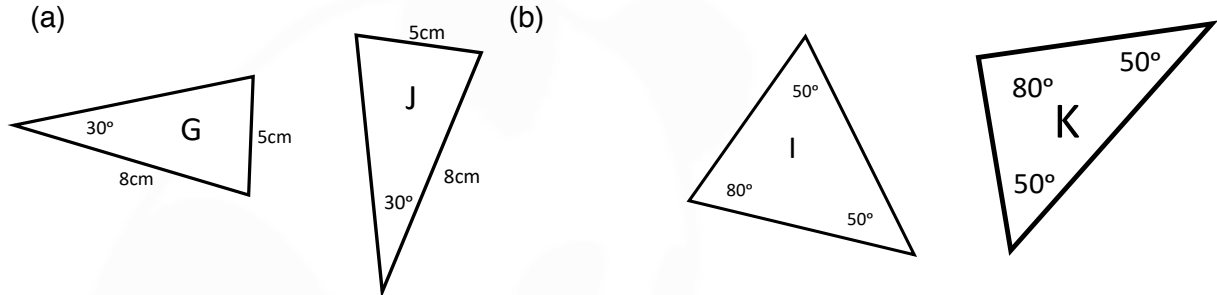


(c)



- Working:**
- (a) Two pairs of corresponding angles are equal and 1 corresponding pair of sides are equal so ASA
  - (b) Two pairs of corresponding sides and the angle in between the sides are equal so SAS
  - (c) All three corresponding sides are equal so SSS

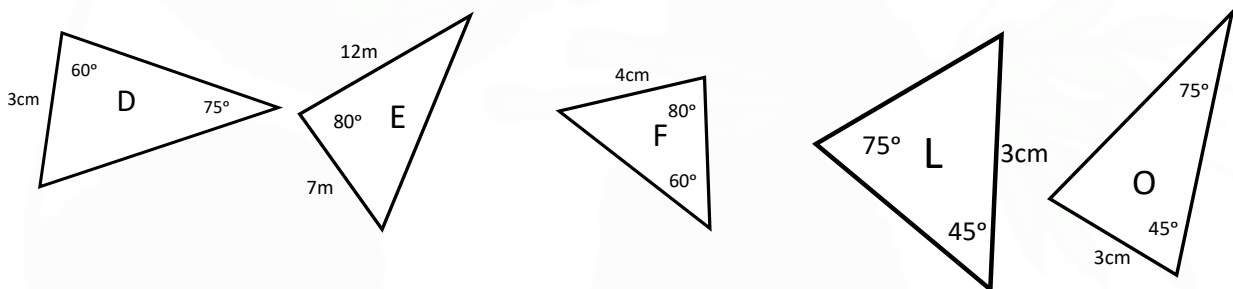
**E.g. 2** Explain why these pairs of triangles are not necessarily congruent.



- Working:**
- (a) It looks like SAS but the angle is not between the two sides
  - (b) AAA is not a condition for triangle congruency — one triangle could be an enlargement of the other.

**E.g. 3** Find the congruent triangles from these triangles.

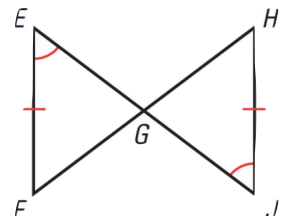
**Hint:** you may need to work out the 3rd angle



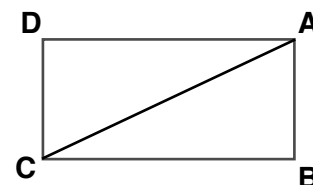
- Working:** L and O are congruent due to ASA — the 3 cm side is opposite to  $75^\circ$  and the  $45^\circ$  angle is next to the 3 cm side  
 Now consider triangle D. The 3rd angle is  $45^\circ$ . So again 3 cm is opposite  $75^\circ$  and the  $45^\circ$  angle is next to the 3cm side. So D is congruent to L and O.

**E.g. 4** Prove that triangle EFG is congruent to GHJ.

- Working:**
- Side:  $EF = HJ$  (given)
  - Angle:  $\widehat{FEG} = \widehat{GJH}$  (given)
  - Angle:  $\widehat{EGF} = \widehat{HGJ}$  (vertically opposite angles)
- Since we have ASA, the triangles are congruent



**E.g. 5** Prove that the triangle ABC is congruent to ACD in the rectangle.



**Working:** No angles or sides are given but we can use the properties of a rectangle to complete the proof

Right-angle:  $\hat{D} = \hat{B} = 90^\circ$  (vertex of a rectangle)

Hypotenuse: AC is common to both triangles and is the hypotenuse

Side: CD = AB (opposite sides in a rectangle are equal length)

Since we have RHS, the triangles ABC and ACD are congruent

**OR**

Angle:  $\angle DCA = \angle BAC$  (alternate angles since DC and AB are parallel)

Side: AD = BC (opposite sides in a rectangle are equal length)

Angle:  $\angle CAD = \angle ACB$  (alternate angles since DA and CB are parallel)

Since we have ASA, the triangles ABC and ACD are congruent

Similar arguments could be used for SAS and SSS.

**Video:** [Congruent triangles](#)  
[Congruent and similar shapes](#)

[Solutions to Starter and E.g.s](#)

### Exercise

9-1 class textbook: p293 M9.7 Qu 1-9

A\*-G class textbook: p256 E9.3 Qu 1-8

9-1 homework book: p100 M9.7 Qu 1-5, 8

A\*-G homework book: p73 E9.3 Qu 1-5, 8

**or** [Congruent Triangles](#) Page 1: Qu 1, 2 **and** Page 2 Apply Qu 1-5

[Homework book answers \(only available during a lockdown\)](#)

[Answers](#)

[Congruent Triangles ANSWERS](#)