Connecting Area and Volume Factors

Starter

1. Given that the area factor is 25, find the:
   (a) length factor (b) the volume factor

   Working: (a) Length factor $= \sqrt{\text{Area factor}} = \sqrt{25} = 5$

   (b) Volume factor $= \text{Length factor}^3 = 5^3 = 125$

2. Given that the volume factor is 27, find the area factor.

   Working: Find the length factor first.
   Length factor $= \sqrt[3]{\text{Volume factor}} = \sqrt[3]{27} = 3$
   Area factor $= \text{Length factor}^2 = 3^2 = 9$

3. The volume ratio is $8 : 125$. Find the: (a) length ratio (b) area ratio.

   Working: (a) Length factor $= \sqrt[3]{\text{Volume factor}}$ so cube root each value
   $\sqrt[3]{8} : \sqrt[3]{125}$
   i.e. $2 : 5$

   (b) Area factor $= \text{Length factor}^2$ so square each value
   $2^2 : 5^2$
   i.e. $4 : 25$

E.g. 1 The cross-section of a hexagonal prism has an area of 18cm^2. A similar prism has a cross-sectional area of 162 cm^2. If the volume of the first prism is 270 cm^3, what is the volume of the second prism?

Working: Enter the information we know in the table

	Area	Volume
Small	18	270
Big	162	*x*

To find x we go towards the unknown so ***small to big***
We need the volume factor, but ***first we calculate the length factor***

Area factor small to big $= \dfrac{162}{18} = 9$ **small to big so > 1**

Length factor small to big $= \sqrt{9} = 3$
Volume factor small to big $= 3^3 = 27$
Volume of second prism, $x = 27 \times 270 = 7290$ cm^3.

E.g. 2 A container has a surface area of 5000 cm² and a capacity of 12.8 litres. Find the surface area of a similar container which has a capacity of 5.4 litres.

Working: Enter the information we know in the table

	Area	Volume
Big	5000	12.8
Small	x	5.4

To find x we go towards the unknown so ***big to small***
We need the area factor, but ***first we calculate the length factor***

Volume factor big to small $= \dfrac{5.4}{12.8}$ **big to small so < 1**

Length factor big to small $= \sqrt[3]{\dfrac{5.4}{12.8}} = \dfrac{3}{4}$

Area factor big to small $= \left(\dfrac{3}{4}\right)^2 = \dfrac{9}{16}$

Surface area of container, $x = \dfrac{9}{16} \times 5000 = 2812.5$ cm².

E.g. 3 The masses of two similar objects are 24 kg and 81 kg respectively. If the surface area of the larger object is 540 cm², find the surface area of the smaller object.

Working: Mass is proportional to volume so the "mass factor" will function in the same way as the volume factor.
Enter the information we know in the table

	Area	Mass
Big	540	81
Small	x	24

To find x we go towards the unknown so ***big to small***
We need the area factor, but ***first we calculate the length factor***

Volume factor big to small $= \dfrac{24}{81} = \dfrac{8}{27}$ **big to small so < 1**

Length factor big to small $= \sqrt[3]{\dfrac{8}{27}} = \dfrac{2}{3}$

Area factor big to small $= \left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9}$

Surface area of smaller object, $x = \dfrac{4}{9} \times 540 = 240$ cm².

E.g. 4 Two similar solids have surface areas in the ratio of $49 : 81$. Find the ratios of:

(a) their side lengths

(b) their volumes.

Working:

(a) Area ratio is $49 : 81$

So length ratio is $\sqrt{49} : \sqrt{81}$

i.e. $7 : 9$

(b) Volume ratio is $7^3 : 9^3$

i.e. $343 : 729$

Video: Connecting area and volume factors

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p457 E13.2 Qu 1-12
A*-G class textbook:	p411 E13.7 Qu 1-7
9-1 homework book:	p158 E13.2 Qu 1-8
A*-G homework book:	p116 E13.7 Qu 1-7

Homework book answers (only available during a lockdown)