

## Dependent Probability

### Starter

In this question you will need to draw 2 tree diagrams and answer the same questions twice — once for '*with replacement*' and once for '*without replacement*'.

1. There are 11 discs in a bag — 7 black and 4 white.  
A disc is chosen at random from the bag and the colour is noted.

(a) **With replacement**  
The disc *is* replaced before another disc is chosen.

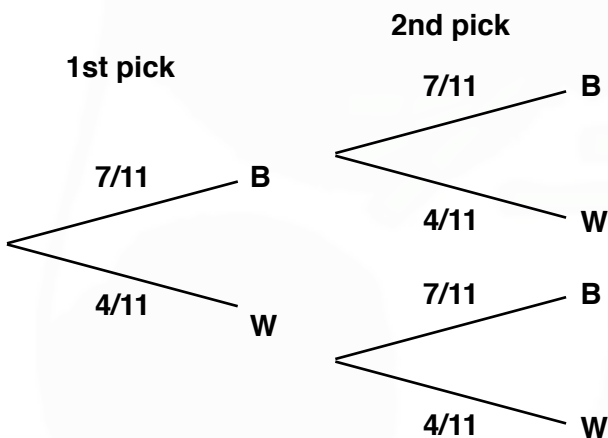
(b) **Without replacement**  
The disc *is not* replaced before another disc is chosen.

### For both situations (a) and (b):

- (i) Draw a tree diagram to represent the outcomes and the probabilities.  
Use your tree diagram to find the probability of getting
- (ii) 2 white discs  
(iii) at least one black disc.  
(iv) one white disc.

### Working:

(a) **With replacement**  
(i)

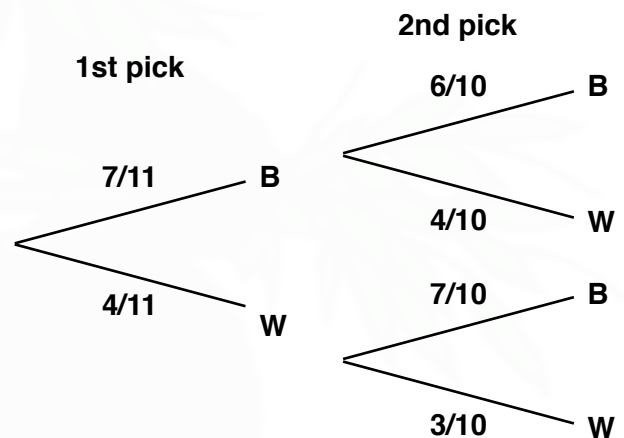


$$(ii) \quad P(W, W) = \frac{4}{11} \times \frac{4}{11} = \frac{16}{121}$$

$$(iii) \quad P(>1 B) = 1 - P(\text{no B}) \\ = 1 - \frac{16}{121} \\ = \frac{105}{121}$$

$$(iv) \quad P(W, B) + P(B, W) = \frac{4}{11} \times \frac{7}{11} + \frac{7}{11} \times \frac{4}{11} \\ = \frac{56}{121}$$

(b) **Without replacement**  
(i)



$$(ii) \quad P(W, W) = \frac{4}{11} \times \frac{3}{10} = \frac{6}{55}$$

$$(iii) \quad P(>1 B) = 1 - P(\text{no B}) \\ = 1 - \frac{6}{55} \\ = \frac{49}{55}$$

$$(iv) \quad P(W, B) + P(B, W) = \frac{4}{11} \times \frac{7}{10} + \frac{7}{11} \times \frac{4}{10} \\ = \frac{28}{55}$$

**E.g. 1** Jack has 8 black socks and 12 green socks in a drawer. He takes out 2 socks at random.

Find the probability that:

- (a) both socks are green
- (b) both socks are the same colour
- (c) at least 1 sock is green

**Working:** (a)  $P(G, G) = \frac{12}{20} \times \frac{11}{19} = \frac{33}{95}$

(b)  $P(G, G) + P(B, B) = \frac{33}{95} + \frac{8}{20} \times \frac{7}{19} = \frac{47}{95}$

(c)  $P(\text{at least 1 sock is green}) = 1 - P(\text{no green socks})$   
 $= 1 - \frac{8}{20} \times \frac{7}{19}$   
 $= \frac{81}{95}$

**E.g. 2** In a school council there are  $n$  students,  $x$  of whom are girls.

- (a) One member of the council is chosen at random. Write down an expression in terms of  $n$  and  $x$  for the probability that the person is:
  - (i) a girl
  - (ii) a boy
- (b) Two members of the council are chosen at random. Write down an expression in terms of  $n$  and  $x$  for the probability that:
  - (i) both members are girls and
  - (ii) both members are the same sex.

**N.B.** For (b) there is no need to expand any brackets.

**Working:** (a) (i)  $P(G) = \frac{x}{n}$

(ii)  $P(B) = \frac{n-x}{n}$

(b) (i)  $P(G, G) = \frac{x}{n} \times \frac{x-1}{n-1} = \frac{x(x-1)}{n(n-1)}$

(ii)  $P(B, B) = \frac{n-x}{n} \times \frac{n-x-1}{n-1} = \frac{(n-x)(n-x-1)}{n(n-1)}$

$P(\text{same sex}) = \frac{x(x-1)}{n(n-1)} + \frac{(n-x)(n-x-1)}{n(n-1)}$

**Video:** [Conditional probability](#)

[Solutions to Starter and E.g.s](#)

### Exercise

9-1 class textbook:	p257 M8.11 Qu 1-10
A*-G class textbook:	p220 E8.1 Qu 1-12
9-1 homework book:	p89 M8.11 Qu 1-5
A*-G homework book:	p63 E8.1 Qu 1-5

[Homework book answers \(only available during a lockdown\)](#)