Direct Proportion Equations (Linear)

Starter

1. **(Review of Y9 material)** If 6 pens cost £25, how much do 20 pens cost?

Working:

6 pens cost £25

Divide by 6***:*** 1 pen costs £$\frac{25}{6}$

Multiply by 20***:*** 20 pens costs $\frac{25}{6} \times 20 = £83.33$

2. **(Review of Y9 material)** How far is 17 km in miles if 8 km ≈ 5 miles?

Working:

8 km ≈ 5 miles

Divide by 8***:*** 1 km $\approx \frac{5}{8}$ miles

Multiply by 17***:*** 17 km $\approx \frac{5}{8} \times 17 = 10.625$ miles

E.g. 1 The number of pens, n, is directly proportional to the cost, C in £.

(a) Given that 8 pens cost £31, find an equation for C in terms of n pen.

(b) How much would 165 pens cost? Give your answer to the nearest pence.

Working:

(a) $C \propto n \Rightarrow C = kn$ ***(write formula with $\propto$ and $=$)***

$C = 31, n = 8$***:*** $31 = 8k$ ***(substitute into the formula)***

$k = 3.875$ ***(solve for k)***

So $C = 3.875n$ ***(replace k by the value found)***

(b) When $n = 165$, $C = 3.875 \times 165$ ***(substitute)***

$C = £639.38$ ***(nearest penny)***

E.g. 2 A quantity P is directly proportional to Q. When $P = 235$, $Q = 5$.

(a) Find the constant of proportionality.

(b) Find P when $Q = 14$.

(c) Find Q when P is 3901.

Working:

(a) $P \propto Q \Rightarrow P = kQ$

$P = 235, Q = 5$: $235 = 5k$

$k = 47$

So $P = 47Q$

(b) ***When $Q = 14$:*** $P = 47 \times 14$

$P = 658$

(c) ***When $P = 3901$:*** $3901 = 47Q$

$Q = \frac{3901}{47}$

$Q = 83$

E.g. 3 The amount of money earned by Sasha, M, is directly proportional to the number of hours, h, she works. If she works for 9.5 hours she earns £155.80.

(a) Express M in terms of h.

(b) Using the equation formed in part (a), find out how many hours it would take her to earn £688.80.

Working:

(a) $M \propto h \quad \Rightarrow \quad M = kh$

$M = 155.80, h = 9.5$: $\quad 155.80 = 9.5k$

$k = 16.4$

So $\quad M = 16.4h$

(b) ***When*** $M = 688.80$: $\quad 688.8 = 16.4h$

$$h = \frac{688.8}{16.4}$$

$h = 42$

Sasha would have to work for 42 hours

E.g. 4 Complete the table given that x and y are directly proportional to one another.

x	7	3		-6	
y	42		84		-54

Working: ***Use the given values to find*** k***:***

$y \propto x \quad \Rightarrow \quad y = kx$

$x = 7, y = 42$: $\quad 42 = 7k$

$k = 6$

So $\quad y = 6x$

The table can now be completed.

x	7	3	14	-6	-9
y	42	18	84	-36	-54

N.B. From the table it can be seen that $\frac{y}{x}$ is constant.

Video: Direct proportion

Solutions to Starter and E.g.s

Exercise

9-1 class textbook: p143 M5.7 Qu 1-11; p145 E5.2 Qu 1, 6

A*-G class textbook: p134 E5.2 Qu 1–7

9-1 homework book: p50 M5.7 Qu 1-8

A*-G homework book: p37 E5.2 Qu 1-5

Homework book answers (only available during a lockdown)