Direct Proportion Equations (Non-Linear)

Starter

1. **(Review of last lesson)** You are given that f is directly proportional to g. When $f = 27$, $g = 378$. What is the value of f when $g = 203$?

Working:

$f \propto g \quad \Rightarrow \quad f = kg$

$f = 27, g = 378: \quad 27 = 378k$

$k = \frac{1}{14}$ (use a fraction to avoid error)

So $f = \frac{1}{14}g$

When $g = 203$, $\quad f = \frac{1}{14} \times 203$

$f = 14.5$

E.g. 1 A quantity A is directly proportional to the square of B. When $A = 36, B = 3$.

(a) Find the constant of proportionality.
(b) Find A when $B = 5$.
(c) Find B when A is 196.

Working:

(a) $A \propto B^2 \quad \Rightarrow \quad A = kB^2$

$A = 36, B = 3 \quad 36 = k \times 3^2$

$k = 4$

(b) $A = 4B^2$

When $B = 5$***:*** $\quad A = 4 \times 5^2$

$A = 100$

(c) A ***is*** $196 \quad 196 = 4B^2$

$49 = B^2$

$B = 7$

N.B. $B = \pm 7$ could be correct.

E.g. 2 Given that $V \propto r^3$, copy and complete the table.

r		2		1.5
V	1.6	32	256	

Hint: Calculate the constant of proportionality so that you have a formula. Substitute numbers into the formula to find the missing values.

Working:

$V \propto r^3 \quad \Rightarrow \quad V = kr^3$

$V = 32, r = 2 \quad 32 = k \times 2^3$

$k = 4$

$V = 4r^3$

Substituting numbers into the formula gives:

r	0.74	2	4	1.5
V	1.6	32	256	13.5

E.g. 3 In an experiment, Julie made measurements of w and p.

w	2	5	7
p	1.6	25	68.6

Which of these laws fits the results: $p \propto w$, $p \propto w^2$ or $p \propto w^3$?

Hint: Calculate the constant of proportionality for all three relationships using one piece of data (e.g. $w = 5$ and $p = 25$ since they are both whole numbers) so that you have a formula for each.
Then substitute the other sets of data points into the formula to see if they give the correct values.

Working:

Assume $p \propto w$ so $p = kw$
Using $w = 5$ ***and*** $p = 25$: $25 = k \times 5$ so $k = 5$
So $p = 5w$
When $w = 2$: $p = 5w = 5 \times 2 = 10 \neq 1.6$
So $p \propto w$ is not the relationship

Assume $p \propto w^2$ so $p = kw^2$
Using $w = 5$ ***and*** $p = 25$: $25 = k \times 5^2$ so $k = 1$
So $p = w^2$
When $w = 7$: $p = w^2 = 7^2 = 49 \neq 68.6$
So $p \propto w^2$ is not the relationship

Assume $p \propto w^3$ so $p = kw^3$
Using $w = 5$ ***and*** $p = 25$: $25 = k \times 5^3$ so $k = \frac{1}{5} = 0.2$
So $p = \frac{1}{5}w^3$
When $w = 2$: $p = \frac{1}{5}w^3 = \frac{1}{5} \times 2^3 = \frac{8}{5} = 1.6$ ✓
When $w = 7$: $p = \frac{1}{5}w^3 = \frac{1}{5} \times 7^3 = \frac{343}{5} = 68.6$ ✓
So $p \propto w^3$ is the relationship

Video: Direct proportion

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p145 E5.2 Qu 2-10
A*-G class textbook:	p135 E5.3 Qu 1-10
9-1 homework book:	p51 E5.2 Qu 1-7
A*-G homework book:	p38 E5.3 Qu 1-7

Homework book answers (only available during a lockdown)