

## Mixed Sequences

### Starter

1. (Review of last lesson)

Find an expression in the form  $u_n = an^2 + bn + c$  for the sequence 3, 9, 17, 27, 39.

**Working:** To find the 2nd differences:

$$\begin{array}{cccccc} 3 & 9 & 17 & 27 & 39 & \\ & 6 & 8 & 10 & 12 & \\ & & 2 & 2 & 2 & \end{array}$$

The second differences are 2  $\Rightarrow a = \frac{1}{2} \times 2 = 1$

$a + b + c = 1\text{st term:}$   $a + b + c = 3$   
 $4a + 2b + c = 2\text{nd term:}$   $4a + 2b + c = 9$

**Replace a by 1:**  $1 + b + c = 3$   
 $4 + 2b + c = 9$

**Rearranging:**

$$\begin{array}{r} b + c = 2 \\ \underline{2b + c = 5} \\ \text{Subtract} \quad -b = -3 \\ \underline{\quad b = 3} \\ \text{Subst.} \quad \quad 3 + c = 2 \\ \underline{\quad \quad c = -1} \end{array}$$

$a = 1, b = 3, c = -1$   
 So  $u_n = 1n^2 + 3n - 1 = n^2 + 3n - 1$

**E.g. 1** Find the n-th term of these sequences:

(a)  $(25 \times 2), (21 \times 4), (17 \times 8), (13 \times 16), \dots$

(b)  $\frac{1}{5}, \frac{4}{7}, \frac{9}{9}, \frac{16}{11}, \dots$

(c)  $(15 \times 1), (10 \times 8), (5 \times 27), (0 \times 64), \dots$

**Working:** (a) The first numbers of the product are: 25, 21, 17, 13, ...  
 Term-to-term rule:  $21 - 25 = -4 \Rightarrow -4n$   
 Term before the first:  $25 - (-4) = 25 + 4 = 29$   
 nth term is  $29 - 4n$

The second numbers of the product are: 2, 4, 8, 16, ...  
 These are the powers of 2 so the nth term is  $2^n$ .

By combining the sequences, we get  $u_n = (29 - 4n) \times 2^n$

(b)  $\frac{1}{5}, \frac{4}{7}, \frac{9}{9}, \frac{16}{11}, \dots$

The numerators of the sequence are: 1, 4, 9, 16, ...

These are the square numbers so the  $n$ th term is  $n^2$

The denominators of the sequence are: 5, 7, 9, 11, ...

Term-to-term rule:  $7 - 5 = 2 \Rightarrow 2n$

Term before the first:  $5 - 2 = 3$

$n$ th term is  $2n + 3$

By combining the sequences, we get  $u_n = \frac{n^2}{2n + 3}$

(c)  $(15 \times 1), (10 \times 8), (5 \times 27), (0 \times 64), \dots$

The first numbers of the product are: 15, 10, 5, 0, ...

Term-to-term rule:  $15 - 10 = -5 \Rightarrow -5n$

Term before the first:  $15 - (-5) = 15 + 5 = 20$

$n$ th term is  $20 - 5n$

The second numbers of the product are: 1, 8, 27, 64, ...

These are the cube numbers so the  $n$ th term is  $n^3$ .

By combining the sequences, we get  $u_n = (20 - 5n) \times n^3$

**Video:** [The  \$n\$ th term for fractional sequences](#)

[Solutions to Starter and E.g.s](#)

### Exercise

9-1 class textbook:

p395 E12.3 Qu 1-7

A\*-G class textbook:

No exercise

9-1 homework book:

p134 E12.3 Qu 1-8

A\*-G homework book:

No exercise

[Homework book answers \(only available during a lockdown\)](#)