

Problems Leading to Quadratics

Starter

1. **(Review of last lesson)** Solve $x - 2 = \frac{4}{x + 1}$.

Working:

$$x - 2 = \frac{4}{x + 1}$$

$$(x - 2)(x + 1) = 4$$

$$x^2 + x - 2x - 2 = 4 \quad \text{expand the brackets}$$

$$x^2 - x - 6 = 0 \quad \text{collect like terms and make } = 0$$

M: $-6 = -3 \times 2$
 A: $-1 = -3 + 2$

$$x^2 - 3x + 2x - 6 = 0 \quad \text{split } -x \text{ into } -3x + 2x$$

$$x(x - 3) + 2(x - 3) = 0 \quad \text{factorise by grouping (same brackets)}$$

$$(x - 3)(x + 2) = 0$$

$$\therefore x - 3 = 0 \quad \text{or} \quad x + 2 = 0$$

So $x = 3$ or $x = -2$

2. I think of a number, square it and then subtract the original number. The result is 30. What was the original number I thought of?

Working:

I think of a number, square it...: x^2
 ...and then subtract the original number: $x^2 - x$
 The result is 30: $x^2 - x = 30$

$$x^2 - x - 30 = 0 \quad \text{make sure the equation } = 0$$

M: $-30 = -6 \times 5$
 A: $-1 = -6 + 5$

$$x^2 - 6x + 5x - 30 = 0 \quad \text{split } -x \text{ into } -6x + 5x$$

$$x(x - 6) + 5(x - 6) = 0 \quad \text{factorise by grouping (same brackets)}$$

$$(x - 6)(x + 5) = 0$$

$$\therefore x - 6 = 0 \quad \text{or} \quad x + 5 = 0$$

So $x = 6$ or $x = -5$

E.g. 1 The dimensions of a rectangle are x m and $(x + 3)$ m. Given that the area is 28 m^2 , find x .

Working:

Length \times width = Area of a rectangle
 $x(x + 3) = 28$

$$x^2 + 3x - 28 = 0 \quad \text{expand the brackets and make } = 0$$

M: $-28 = -4 \times 7$
 A: $3 = -4 + 7$

$$x^2 - 4x + 7x - 28 = 0 \quad \text{split } 3x \text{ into } -4x + 7x$$

$$x(x - 4) + 7(x - 4) = 0 \quad \text{factorise by grouping (same brackets)}$$

$$(x - 4)(x + 7) = 0$$

$$\therefore x - 4 = 0 \quad \text{or} \quad x + 7 = 0$$

$x = 4$ or $x = -7$
 Since $x > 0$, $x = 4$

E.g. 2 The base of a triangle is x and its perpendicular height is $x - 5$. If the area is 12 cm^2 , find x .

Working:

$$\frac{1}{2} \times \text{base} \times \text{perpendicular height} = \text{area of triangle}$$
$$\therefore \frac{1}{2}x(x - 5) = 12$$
$$x^2 - 5x = 24$$
$$x^2 - 5x - 24 = 0$$

*multiply by 2 and expand the brackets
make sure the equation = 0*

$$\begin{array}{l} \text{M:} \quad -24 = -8 \times 3 \\ \text{A:} \quad \quad 5 = -8 + 3 \end{array}$$
$$x^2 - 8x + 3x - 24 = 0$$
$$x(x - 8) + 3(x - 8) = 0$$
$$(x - 8)(x + 3) = 0$$
$$\therefore x - 8 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = 8 \quad \text{or} \quad x = -3$$

Since $x > 0$, $x = 8$

*split $-5x$ into $-8x + 3x$
factorise by grouping (same brackets)*

E.g. 3 A right-angle triangle has sides x , $x + 2$ and $x + 4$. Find x .

Working:

$x + 4$ is the longest side so it must be the hypotenuse.

$$x^2 + (x + 2)^2 = (x + 4)^2$$

by Pythagoras

$$x^2 + x^2 + 2x + 2x + 4 = x^2 + 4x + 4x + 16$$

expand the brackets

$$x^2 - 4x - 12 = 0$$

collect like terms and make = 0

$$\begin{array}{l} \text{M:} \quad -12 = -6 \times 2 \\ \text{A:} \quad \quad -4 = -6 + 2 \end{array}$$
$$x^2 - 6x + 2x - 12 = 0$$
$$x(x - 6) + 2(x - 6) = 0$$
$$(x - 6)(x + 2) = 0$$
$$\therefore x - 6 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 6 \quad \text{or} \quad x = -2$$

Since $x > 0$, $x = 6$

*split $-4x$ into $-6x + 2x$
factorise by grouping (same brackets)*

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook:	p117 E4.5 Qu 1-3, 5-15
A*-G class textbook:	p107 E4.5 Qu 1-3, 5-15
9-1 homework book:	p42 E4.5 Qu 1-9
A*-G homework book:	p30 E4.5 Qu 1-7

[Homework book answers \(only available during a lockdown\)](#)