

Quadratic Sequences

Starter

1. **(Review of last lesson)**

Find the formula for the n th term of the linear sequence given by $u_1 = 8, u_{n+1} = u_n + 3$.

Working:

$$u_1 = 8$$

$$u_2 = u_1 + 3 = 8 + 3 = 11$$

$$u_3 = u_2 + 3 = 11 + 3 = 14$$

Term-to-term rule: $11 - 8 = 3 \Rightarrow 3n$

Term before the first: $8 - 3 = 5$

$\therefore n$ th term, $u_n = 3n + 5$

2. **(Review of previous material)**

Find the second differences for the sequence 4, 7, 12, 19, 28.

Working:

4	7	12	19	28
	3	5	7	9
		2	2	2

The second differences are 2.

3. Consider the expression $u_n = an^2 + bn + c$. Find expressions in terms of a, b and c for:

(a) u_1

(b) u_2

Working: (a) $u_1 = a \times 1^2 + b \times 1 + c = a + b + c$

(b) $u_2 = a \times 2^2 + b \times 2 + c = 4a + 2b + c$

4. **(Review of previous material)** Solve these simultaneous equations:

(a) $b + c = 7$
 $2b + c = 12$

(b) $b + c = 8$
 $2b + c = 16$

Working: (a)

$$\begin{array}{r} b + c = 7 \\ 2b + c = 12 \\ \text{Sub. } -b = -5 \\ \hline b = 5 \\ \text{Subst. } 5 + c = 7 \\ \hline c = 2 \\ b = 5, c = 2 \end{array}$$

(b)

$$\begin{array}{r} b + c = 8 \\ 2b + c = 16 \\ \text{Add } -b = -8 \\ \hline b = 8 \\ \text{Subst. } 8 + c = 8 \\ \hline c = 0 \\ b = 8, c = 0 \end{array}$$

E.g. 1 Find an expression for the n th term of the quadratic sequence 4, 7, 12, 19, 28.

Working: From the starter the second differences are 2 $\Rightarrow a = \frac{1}{2} \times 2 = 1$

$$\begin{array}{l} a + b + c = \text{1st term:} \quad a + b + c = 4 \\ 4a + 2b + c = \text{2nd term:} \quad 4a + 2b + c = 7 \end{array}$$

Replace a by 1:

$$\begin{array}{l} 1 + b + c = 4 \\ 4 + 2b + c = 7 \end{array}$$

Rearranging:

$$\begin{array}{r} b + c = 3 \\ 2b + c = 3 \\ \text{Subtract} \quad -b \quad = 0 \\ \hline b = 0 \\ \text{Subst.} \quad 0 + c = 3 \\ \hline c = 3 \end{array}$$

$$\begin{array}{l} a = 1, b = 0, c = 3 \\ \text{So } u_n = 1n^2 + 0n + 3 = n^2 + 3. \end{array}$$

E.g. 2 Find an expression in the form $u_n = an^2 + bn + c$ for the sequence 0, 5, 12, 21, 32.

Working: **To find the 2nd differences:**

$$\begin{array}{cccccc} 0 & 5 & 12 & 21 & 32 & \\ & 5 & 7 & 9 & 11 & \\ & & 2 & 2 & 2 & \end{array}$$

The second differences are 2 $\Rightarrow a = \frac{1}{2} \times 2 = 1$

$$\begin{array}{l} a + b + c = \text{1st term:} \quad a + b + c = 0 \\ 4a + 2b + c = \text{2nd term:} \quad 4a + 2b + c = 5 \end{array}$$

Replace a by 1:

$$\begin{array}{l} 1 + b + c = 0 \\ 4 + 2b + c = 5 \end{array}$$

Rearranging:

$$\begin{array}{r} b + c = -1 \\ 2b + c = 1 \\ \text{Subtract} \quad -b \quad = -2 \\ \hline b = 2 \\ \text{Subst.} \quad 2 + c = -1 \\ \hline c = -3 \end{array}$$

$$\begin{array}{l} a = 1, b = 2, c = -3 \\ \text{So } u_n = 1n^2 + 2n - 3 = n^2 + 2n - 3 \end{array}$$

E.g. 3 Find an expression for the n th term of the quadratic sequence 6, 13, 24, 39, 58.

Working: *To find the 2nd differences:*

$$\begin{array}{cccccc} 6 & 13 & 24 & 39 & 58 & \\ & 7 & 11 & 15 & 19 & \\ & & 4 & 4 & 4 & \end{array}$$

The second differences are 4 $\Rightarrow a = \frac{1}{2} \times 4 = 2$

$a + b + c = 1\text{st term:}$ $a + b + c = 6$
 $4a + 2b + c = 2\text{nd term:}$ $4a + 2b + c = 13$

Replace a by 2: $2 + b + c = 6$
 $8 + 2b + c = 13$

Rearranging:

$$\begin{array}{r} b + c = 4 \\ 2b + c = 5 \\ \text{Subtract} \quad \underline{-b} = -1 \\ \qquad \qquad \qquad b = 1 \\ \text{Subst.} \quad \underline{1 + c = 4} \\ \qquad \qquad \qquad c = 3 \end{array}$$

$a = 2, b = 1, c = 3$
 So $u_n = 2n^2 + 1n + 3 = 2n^2 + n + 3$

E.g. 4 Find an expression for the n th term of the quadratic sequence 5, 8, 15, 26, 41.

Working: *To find the 2nd differences:*

$$\begin{array}{cccccc} 5 & 8 & 15 & 26 & 41 & \\ & 3 & 7 & 11 & 15 & \\ & & 4 & 4 & 4 & \end{array}$$

The second differences are 4 $\Rightarrow a = \frac{1}{2} \times 4 = 2$

$a + b + c = 1\text{st term:}$ $a + b + c = 5$
 $4a + 2b + c = 2\text{nd term:}$ $4a + 2b + c = 8$

Replace a by 2: $2 + b + c = 5$
 $8 + 2b + c = 8$

Rearranging:

$$\begin{array}{r} b + c = 3 \\ 2b + c = 0 \\ \text{Subtract} \quad \underline{-b} = 3 \\ \qquad \qquad \qquad b = -3 \\ \text{Subst.} \quad \underline{-3 + c = 3} \\ \qquad \qquad \qquad c = 6 \end{array}$$

$a = 2, b = -3, c = 6$
 So $u_n = 2n^2 - 3n + 6$

- Video:** [Quadratic sequences \(method 1\)](#)
- Video:** [Quadratic sequences \(method 2\)](#)
- Video:** [Quadratic sequences \(method 3\)](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook: p394 E12.2 Qu 1-11
A*-G class textbook: p355 E12.1 Qu 1-12
9-1 homework book: p133 E12.2 Qu 1-5
A*-G homework book: p100 E12.1 Qu 1-5

[Homework book answers \(only available during a lockdown\)](#)

