

Trial and Improvement

Starter

1. (Review of last lesson)

I think of a number, square it and then add five times the original number. The result is 24.

Working: I think of a number, square it...: x^2
 ...and then add five times the original number: $x^2 + 5x$
 The result is 24: $x^2 + 5x = 24$
 $x^2 + 5x - 24 = 0$ *make sure the equation = 0*
 M: $-24 = -3 \times 8$
 A: $5 = -3 + 8$
 $x^2 - 3x + 8x - 24 = 0$ *split 5x into -3x + 8x*
 $x(x - 3) + 8(x - 3) = 0$ *factorise by grouping (same brackets)*
 $(x - 3)(x + 8) = 0$
 $\therefore x - 3 = 0$ or $x + 8 = 0$
 So $x = 3$ or $x = -8$

E.g. 1 Locate the root of the equation $x^3 + 5x = 175$ between two consecutive integers.

Working: Rearrange the equation so that $f(x) = 0$: $x^3 + 5x - 175 = 0$
 So $f(x) = x^3 + 5x - 175$
 $f(2) = 2^3 + 5 \times 2 - 175 < 0$ *actual values are not required*
N.B. $f(2)$ means replace x by 2 in the expression
 $f(4) = 4^3 + 5 \times 5 - 175 < 0$
 $f(5) = 5^3 + 5 \times 5 - 175 < 0$
 $f(6) = 6^3 + 5 \times 6 - 175 > 0$ *change of sign*
 So the solution of $x^3 + 5x - 175 = 0$ lies between 5 and 6

E.g. 2 Locate the root of the equation $x^3 = 51 - 2x$ between two consecutive integers.

Working: Rearrange the equation so that $f(x) = 0$: $x^3 + 2x - 51 = 0$
 So $f(x) = x^3 + 2x - 51$
 $f(2) = 2^3 + 2 \times 2 - 51 < 0$ *actual values are not required*
N.B. $f(2)$ means replace x by 2 in the expression
 $f(3) = 3^3 + 2 \times 3 - 51 < 0$
 $f(4) = 4^3 + 2 \times 4 - 51 > 0$ *change of sign*
 So the solution of $x^3 + 2x - 51 = 0$ lies between 3 and 4

E.g. 3 Solve the equation $x^3 + 5x = 175$ to 1 d.p.

Working: Rearrange the equation so that $f(x) = 0$: $x^3 + 5x - 175 = 0$
 So $f(x) = x^3 + 5x - 175$
 From **E.g. 1**, we know that $f(5) < 0$ (so $x = 5$ is too small) and $f(6) > 0$ (so $x = 6$ is too big)
N.B. Too big (or too small) means that the x -value is too big (or too small)

x -value	$f(x)$	Too big/small
5		Too small
6		Too big
5.5	$5.5^3 + 5 \times 5.5 - 175 = 18.875$	Too big
5.3	$5.3^3 + 5 \times 5.3 - 175 = 0.377$	Too big
5.2	$5.2^3 + 5 \times 5.2 - 175 = -8.392$	Too small

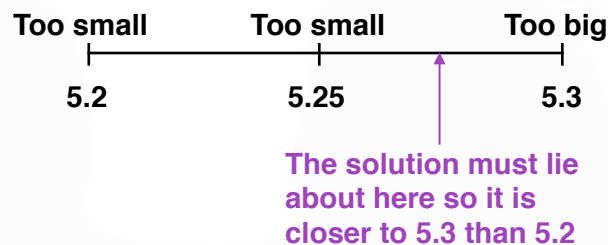
At this point we that the root is between 5.2 and 5.3.

It looks like it is closer to 5.3 than to 5.2 but must do one more calculation to confirm this.

Our final calculation must be *half-way between the consecutive values* to required accuracy — in this case half-way between 5.2 and 5.3.

$$5.25^3 + 5 \times 5.25 - 175 = -4.04... \quad \text{i.e. too small}$$

This diagram can help you to decide whether it is 5.2 or 5.3:



The solution to $x^3 + 5x = 175$ is 5.3 to 1 d.p.

E.g. 4 Solve the equation $x^3 = 51 - 2x$ to 1 d.p.

Working: Rearrange the equation so that $f(x) = 0$: $x^3 + 2x - 51 = 0$
So $f(x) = x^3 + 2x - 51$
From **E.g. 2** we already know that $f(3) < 0$ (so $x = 3$ is too small)
and $f(4) > 0$ (so $x = 4$ is too big)

x -value	$f(x)$	Too big/small
3	$3^3 + 2 \times 3 - 51 = -18$	Too small
4	$4^3 + 2 \times 4 - 51 = 21$	Too big
3.5	$3.5^3 + 2 \times 3.5 - 51 = -1.125$	Too small
3.6	$3.6^3 + 2 \times 3.6 - 51 = 2.856$	Too big
3.55	$3.55^3 + 2 \times 3.55 - 51 \approx 0.839$	Too big

Since $x = 3.5$ is too small and $x = 3.55$ is too big, the answer must be $x = 3.5$ to 1 d.p.

E.g. 5 A solution to $x^3 - 3x = 170$ lies between 5 and 6. Find the solution correct to 1 d.p.

Working: *Rearrange the equation so that $f(x) = 0$:* $x^3 - 3x - 170 = 0$
So $f(x) = x^3 - 3x - 170$
We are given that the solution is between 5 and 6 these are the first values substituted into $f(x)$

x -value	$f(x)$	Too big/small
5	$5^3 - 3 \times 5 - 170 = -60$	Too small
6	$6^3 - 3 \times 6 - 170 = 28$	Too big
5.7	$5.7^3 - 3 \times 5.7 - 170 = -1.907$	Too small
5.8	$5.8^3 - 3 \times 5.8 - 170 = 7.712$	Too big
5.75	$5.75^3 - 3 \times 5.75 - 170 \approx 2.86$	Too big

Since $x = 5.7$ is too small and $x = 5.75$ is too big, the answer must be $x = 5.7$ to 1 d.p.

E.g. 6 A solution to $2x^3 + 7x = 100$ lies between 3.3 and 3.4. Find the solution correct to **2 d.p.**

Working: *Rearrange the equation so that $f(x) = 0$:* $2x^3 + 7x - 100 = 0$
So $f(x) = 2x^3 + 7x - 100$
We are given that the solution is between 3.3 and 3.4 these are the first values substituted into $f(x)$

x -value	$f(x)$	Too big/small
3.3	$2 \times 3.3^3 + 7 \times 3.3 - 100 \approx -5.0$	Too small
3.4	$2 \times 3.4^3 + 7 \times 3.4 - 100 \approx 2.4$	Too big
3.35	$2 \times 3.35^3 + 7 \times 3.35 - 100 \approx -1.4$	Too small
3.37	$2 \times 3.37^3 + 7 \times 3.37 - 100 \approx 0.14$	Too big
3.36	$2 \times 3.36^3 + 7 \times 3.36 - 100 \approx -0.61$	Too small
3.365	$2 \times 3.365^3 + 7 \times 3.365 - 100 \approx -0.24$	Too small

Since $x = 3.365$ is too small and $x = 3.37$ is too big, the answer must be $x = 3.37$ to 2 d.p.

Video: [Trial and improvement](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook:

p179 E6.4 Qu 1-9

A*-G class textbook:

p155 M6.5 Qu 1-12

9-1 homework book:

p64 E6.4 Qu 1-4

A*-G homework book:

p43 M6.5 Qu 1-4

[Homework book answers \(only available during a lockdown\)](#)