Venn Diagrams and Conditional Probability

Starter

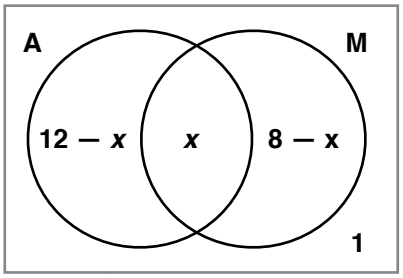
1. In a group of 16 students, 12 take art and 8 take music. One student takes neither art nor music.
   (a) Draw a Venn diagram to help you find the number of students who take both art and music.
   (b) A student is selected at random. Find the probability the student takes art but not music.
   (c) A ***music*** student is chosen at random. Find the probability that they do not study art.
   (d) An ***art*** student is chosen at random. Find the probability that they also study music.

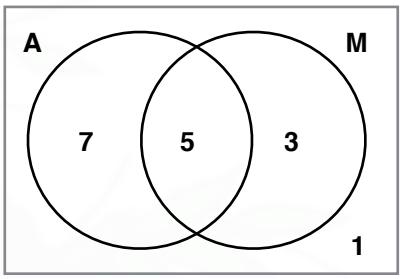
Working:

(a) 1 goes outside both circles
Let x be the overlap

$12 - x$ in A but outside M
$8 - x$ in M but outside A

16 students in total so:
$12 - x + x + 8 - x + 1 = 16$
$x = 5$
So 5 students take art and music





(b) 7 students take A but not M
So $P(\text{A but not M}) = \frac{7}{16}$

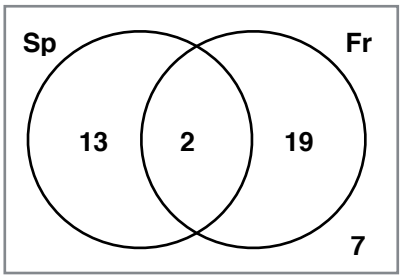
(c) "A ***music*** student is chosen"
Only consider the M circle
5 + 3 = 8 students in total ***this is the denominator***
Do not study art = 3

$$P(\text{study M but not A given study M}) = \frac{3}{5+3} = \frac{3}{8}$$

(d) "An ***art*** student is chosen" so only consider the A circle
7 + 5 = 12 students in total ***this is the denominator***
Also study music = 5

$$P(\text{study A and M given study A}) = \frac{5}{7+5} = \frac{5}{12}$$

E.g. 1 The Venn diagram shows the number of students studying Spanish and French at a school.



(a) A student is chosen at random. Find the probability he/she studies French.

(b) A student is chosen at random. Find the probability he/she does not study Spanish.

(c) A French student is chosen at random. Find the probability he/she studies Spanish.

(d) A Spanish student is chosen at random. Find the probability he/she does not study French.

Working:

(a) "A student is chosen at random." ≡ all students considered (41)

$$P(\text{studies French}) = \frac{2+19}{13+2+19+7} = \frac{21}{41}$$

(b) "A student is chosen at random." ≡ all students considered (41)

$$P(\text{not Spanish}) = \frac{19+7}{41} = \frac{26}{41}$$

(c) "A French student is chosen at random." ≡ only F is considered

Total French students = 2 + 19 = 21

Also study Spanish = 2

$$P(\text{studies Spanish given study French}) = \frac{2}{2+19} = \frac{2}{21}$$

(d) "A Spanish student is chosen at random." ≡ only S is considered

Total Spanish students = 13 + 2 = 15

Not study French = 13

$$P(\text{not French given study Spanish}) = \frac{13}{2+13} = \frac{13}{15}$$

E.g 2* There are 120 students in a school. 40 study Chinese (C), 35 study Japanese (J), and 30 study Spanish (S).

(a) Complete the Venn diagram by filling in the numbers in the missing spaces, including outside the circles.

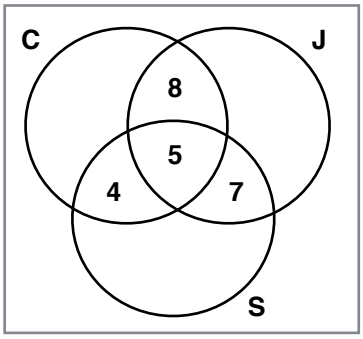
(b) A student is chosen at random. Find the probability that they:

(i) study Chinese, Japanese and Spanish

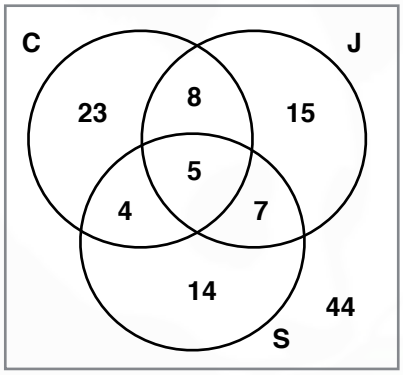
(ii) Chinese and Japanese

(iii) Spanish and Japanese but not Chinese

(iv) study Japanese given that they study Chinese.



Working: (a)



(b) (i) $\frac{5}{120} = \frac{1}{24}$ ***intersection of all 3 circles***

(ii) $\frac{13}{120}$ ***intersection of C and J circles***

(iii) $\frac{7}{120}$ ***intersection of S and J circles, excluding C***

(iv) $P(\text{Japanese given study Chinese}) = \frac{5+8}{4+5+8+23}$

$= \frac{13}{40}$

Video: Venn diagrams

Solutions to Starter and E.g.s

Exercise

Worksheet **Solving problems using Venn diagrams** Qu 4-11

9-1 class textbook: p246 M8.7 Qu 1-6 (Look at Qu 2 and 5 in class)

A*-G class textbook: No exercise

9-1 homework book: p84 M8.7 Qu 1-5

A*-G homework book: No exercise

Solving problems using Venn diagrams SOLUTIONS

Homework book answers (only available during a lockdown)