Volume and Surface Area of a Cylinder

Starter

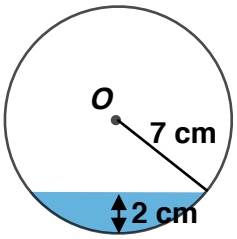
1. **(Review of last lesson)**

A pipe of radius 7 cm can have a maximum depth of 2 cm.

(a) Find the shaded area (cross-sectional area of water).

(b) What is the volume of water in a length of 30 cm of the pipe?

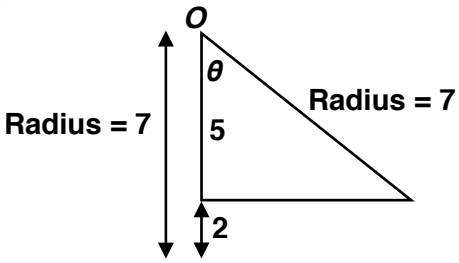
Hint: Find the distance from O to the surface of the water.



Working: (a) The perpendicular distance from O to the surface of water is $7 - 2 = 5$

$$\therefore \cos\theta = \frac{5}{7}$$

$$\theta = \cos^{-1}\frac{5}{7} \approx 44.42^\circ$$



So the angle at the centre is $2\theta = 2\cos^{-1}\frac{5}{7} \approx 88.83^\circ$

Area of segment = Area of sector − Area of triangle

$$= \frac{88.83}{360} \times \pi \times 7^2 - \frac{1}{2} \times 7 \times 7 \times \sin 44.42$$

$$= 13.5 \text{ cm}^2 \text{ (3 s.f.)}$$

(b) Volume of water $= 13.489... \times 30 = 404.7$ cm³ (4 s.f.)

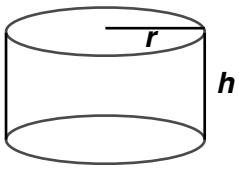
Notes

Volume of a cylinder

A cylinder is a prism with a circular cross-section. The volume of a prism is given by:

Volume of prism = Area of cross-section × Length

E.g. 1 Using the formula for the prism, write down the formula for the volume of a cylinder whose radius is r and whose height is h.



Working: Area of cross-section $= \pi r^2$

So $V = \pi r^2 h$

Volume of a cylinder, $V = \pi r^2 h$

E.g. 2 Calculate the volume of a cylinder whose height is 10 cm and whose diameter is 12 cm. Give your answer in terms of π.

Working: Since the diameter is 12 cm, the radius is 6 cm.

Volume of a cylinder, $V = \pi \times 6^2 \times 10 = 360\pi$ cm³

E.g. 3 A gold cube of side 5 cm is melted down to form a cylinder whose height is 3 cm. Find the cylinder's radius.

Working:

Volume of cube $= 5^3 = 125$

$$\therefore \pi \times r^2 \times 3 = 125$$

$$r^2 = \frac{125}{3\pi}$$

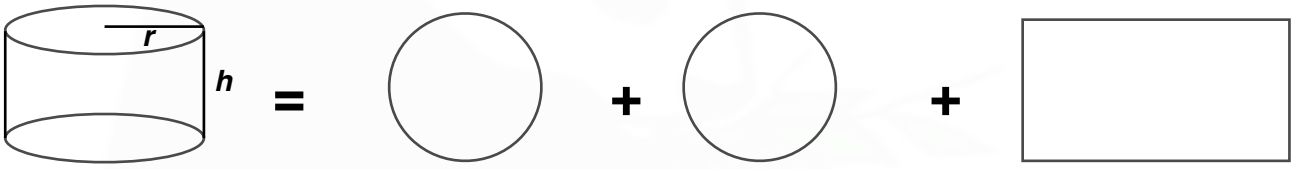
$$r = \sqrt{\frac{125}{3\pi}} = 3.64 \text{ cm}$$

Surface area of a cylinder

What shapes is the surface area of a cylinder made up of?

2 circles and a rectangle

E.g. 4 Using the diagram below, find a formula for the surface area of a cylinder.



Working:

The length of the rectangle is equal to the circumference of the circle.

Surface area = Area of circle + Area of circle + Area of rectangle

$$= \quad \pi r^2 \quad + \quad \pi r^2 \quad + \quad 2\pi r \times h$$

$$= 2\pi r^2 + 2\pi rh$$

Surface area of cylinder, $SA = 2\pi r^2 + 2\pi rh$

E.g. 5 Find the total surface area of a cylinder whose radius is 3 cm and whose height is 8 cm. Give your answer in terms of π.

Working:

Surface area $= 2\pi r^2 + 2\pi rh$

$= 2\pi \times 3^2 + 2\pi \times 3 \times 8$

$= 18\pi + 48\pi$

$= 66\pi$ cm² (4 s.f.)

E.g. 6 A cylinder has total surface area of 110π. Find its height given that its radius is 5 cm.

Working:

$$2\pi \times 5^2 + 2\pi \times 5 \times h = 110\pi$$

$$50\pi + 10\pi h = 110\pi$$

$$10\pi h = 60\pi$$

$$h = \frac{60\pi}{10\pi} = 6 \text{ cm}$$

E.g. 7 A label is to cover the curved surface of a baked beans tin. The tin has a diameter of 74 mm and a height of 115 mm. Given that the label will overlap by 1 cm, find the area of the label in:
(a) mm² (b) cm².

Working: (a) The label is a rectangle whose height is 115 mm
The length of the rectangle is the circumference plus 1 cm
Length $= 74\pi + 10$ $C = \pi d$ **and** 1 **cm** $\equiv 10$ **mm**
Area of label $= 115(74\pi + 10) = 27885$ mm² (5 s.f.) and

(b) 1 cm $\equiv 10$ mm so 1 cm² $\equiv 10^2$ mm² $= 100$ mm²
27885 mm² $\equiv 278.85$ cm² (2 d.p.)

Video: Volume of a cylinder
Video: Surface area of a cylinder

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p438 M13.5 Qu 7-14
A*-G class textbook:	p393 M13.2 Qu 7-14
9-1 homework book:	p150 M13.5/13.6 Qu 6-12
A*-G homework book:	p110 M13.2 Qu 6-10

Homework book answers (only available during a lockdown)