

## Volume of Spheres, Pyramids and Cones

### Starter

1. **(Review of last lesson)** The surface area of a cylinder is  $80\pi$  cm<sup>2</sup>. Given that the height of the cylinder is 3 cm, calculate its radius.

**Working:** Surface area =  $2\pi r^2 + 2\pi rh$

**Substituting:**  $2\pi r^2 + 2\pi r \times 3 = 80\pi$

**Dividing by  $2\pi$ :**  $r^2 + 3r = 40$

$$r^2 + 3r - 40 = 0$$

**Factorising:**  $1 \times -40 = -40 \Rightarrow$  Multiply:  $-40 = -5 \times 8$

Add:  $3 = -5 + 8$

**Split  $3r$  into  $-5r + 8r$ :**  $r^2 + 3r - 40 = r^2 - 5r + 8r - 40$

**Factorise by grouping (same brackets):**  $= r(r - 5) + 8(r - 5)$

$$= (r - 5)(r + 8)$$

$$\therefore (r - 5)(r + 8) = 0$$

So  $r - 5 = 0$  or  $r + 8 = 0$

Since  $r > 0$ ,  $r = 5$

The radius of the cylinder is 5.

- E.g. 1** Find the volume of sphere whose radius is 5 cm.

**Working:** Volume of the sphere =  $\frac{4}{3}\pi \times 5^3 = \frac{500\pi}{3} = 523.6$  cm<sup>3</sup>

- E.g. 2** Find the exact volume of a cone whose base radius is 5 cm and whose perpendicular height is 12 cm. Give your answer in terms of  $\pi$ .

**Working:** Volume of the cone =  $\frac{1}{3}\pi \times 5^2 \times 12 = 100\pi$  cm<sup>3</sup>

**E.g. 3** A cone of height 6 cm is removed the top of a larger cone to form a frustum. The radius of the top of the frustum is 4 cm and the radius of the base of the frustum is 12 cm. Calculate the volume of the frustum.

**Working:** The radii of the bases of the removed cone and original cone are 4 and 12

$$\text{So the length factor} = \frac{12}{4} = 3$$

The original cone is three times taller than the removed cone

$$\text{So height of original cone} = 3 \times 6 = 18$$

Volume of frustum = Volume of original cone – Volume of removed cone

$$= \frac{1}{3}\pi \times 12^2 \times 18 - \frac{1}{3}\pi \times 4^2 \times 6$$

$$= 864\pi - 32\pi$$

$$= 832\pi$$

**Alternatively:** Length factor =  $\frac{12}{4} = 3$

$$\text{Volume factor} = 3^3 = 27$$

Volume of original cone = 27 × volume of removed cone

$$= 27 \times \frac{1}{3}\pi \times 4^2 \times 6$$

$$= 27 \times 32\pi$$

$$= 864\pi$$

Volume of frustum = Volume of original cone – Volume of removed cone

$$= 864\pi - 32\pi$$

$$= 832\pi$$

**Video:** [Volume of a sphere](#)

**Video:** [Volume of a cone](#)

**Video:** [Volume of a frustum](#)

[Solutions to Starter and E.g.s](#)

### Exercise

9-1 class textbook: p441 M13.7 Qu 1-18

A\*-G class textbook: p396 E13.4 Qu 1-18

9-1 homework book: p152 M13.7 Qu 1-7

A\*-G homework book: p111 E13.4 Qu 1-7

[Homework book answers \(only available during a lockdown\)](#)