

Algebraic proof

Starter

1. (Review of last lesson) Solve $\frac{x+3}{4} = \frac{x+9}{7}$.

2. (Review of last lesson) Solve $\frac{4}{x-2} + \frac{7}{x+1} = 3$, giving your answers to 3 s.f.

Notes

Consecutive integers: ..., $x - 1, x, x + 1, x + 2, \dots$

Even number are of the form $2n$ because they are always divisible by 2.

Odd numbers are of the form $2n + 1$ or $2n - 1$ because they are always 1 more or 1 less than an even number.

Consecutive even numbers: ..., $2n - 2, 2n, 2n + 2, 2n + 4, \dots$

Consecutive odd numbers: ..., $2n - 1, 2n + 1, 2n + 3, 2n + 5, \dots$

Sometimes questions state “two even numbers” — this means you need to have **different** even numbers.

- Two different even number: use $2x$ and $2y$
- Two different odd number: use $2x + 1$ and $2y + 1$

E.g. 1 Prove that the sum of two odd numbers is even.

Working: Let the 2 odd numbers be $2x + 1$ and $2y + 1$
 $2x + 1 + 2y + 1 = 2(x + y + 1)$ — which is in the form of an even number.

E.g. 2 Prove that the answer to every line of the pattern below is 8:

$$3 \times 5 - 1 \times 7 \qquad 4 \times 6 - 2 \times 8 \qquad 5 \times 7 - 3 \times 9$$

Hint: look at how the digits of each calculation are connected to each other.

E.g. 3 If a and b are both odd, prove that ab is odd.

Working: **Remember** an odd number is of the form $2n + 1$
 Let a and b be odd numbers such that $a = 2x + 1$ and $b = 2y + 1$.
 Then $ab = (2x + 1)(2y + 1)$
 $= 4xy + 2x + 2y + 1$
 $= 2(2xy + x + y) + 1$
 which is of the form $2 \times \text{a number} + 1$
 i.e. it is an odd number
 Therefore, the product of 2 odd numbers is always odd

N.B. a and b cannot be $2x + 1$ and $2x + 3$ as this would be 2 **consecutive** odd numbers

E.g. 4 For three consecutive numbers, prove that the product of the first and third number is one less than the square of the middle number.

Video: [Algebraic proof](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook: p525 E16.8 Qu 1-18
A*-G class textbook: p483 E16.5 Qu 1-14
9-1 homework book: p178 E16.8 Qu 1-10
A*-G homework book: p134 E16.5 Qu 1-8

Summary

Consecutive integers: $\dots, x - 1, x, x + 1, x + 2, \dots$

Even number are of the form $2n$.

Odd numbers are of the form $2n + 1$ or $2n - 1$.

Consecutive even numbers: $\dots, 2n - 2, 2n, 2n + 2, 2n + 4, \dots$

Consecutive odd numbers: $\dots, 2n - 1, 2n + 1, 2n + 3, 2n + 5, \dots$

Two different even number: use $2x$ and $2y$

Two different odd number: use $2x + 1$ and $2y + 1$

[Homework book answers \(only available during a lockdown\)](#)