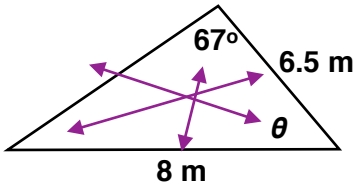
Cosine Rule

Starter

1. **(Review of last lesson)**
Find the value of the missing angle to 3 s.f..



2. Point B is 13 km north of point A. Point C lies 19 km from point B and on a bearing of 052° from A. Find the bearing of C from B, to the nearest degree.

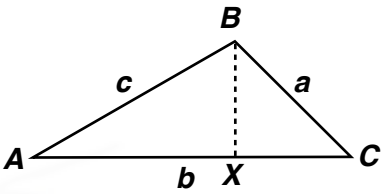
Notes

A close cousin of the sine rule is the cosine rule.

Proof of the cosine rule

Let BX be perpendicular to AC so that $b = AX + CX$

N.B. $b = AC$



We want to find a formula for side a in terms of b, c and A.
To do this, we will use Pythagoras in the $\triangle BCX$.
Before this we have to find CX and BX in terms of b, c and A

Finding CX***:*** Using trigonometry, $\cos A = \dfrac{AX}{c} \quad \Rightarrow \quad AX = c\cos A$

$CX = AC - AX = b - c\cos A$ ***an expression for*** CX ***is ready***

Finding BX***:*** Using Pythagoras, $BX^2 = AB^2 - AX^2$

$= c^2 - (c\cos A)^2$ ***an expression for*** BX ***is ready***

Finding a***:*** Use Pythagoras in $\triangle BCX$:

$BC^2 = BX^2 + CX^2$

$a^2 = c^2 - (c\cos A)^2 + (b - c\cos A)^2$

$a^2 = c^2 - c^2\cos^2 A + b^2 - 2bc\cos A + c^2\cos^2 A$

$a^2 = b^2 + c^2 - 2bc\cos A$ — formula for cosine rule

The above is the formula for finding a side using the cosine rule — what if we want to find an angle?

E.g. 1 Rearrange $a^2 = b^2 + c^2 - 2bc\cos A$ to make $\cos A$ the subject.

Information needed to use the cosine rule

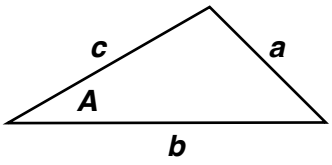
What information do we need to find a side/an angle using the cosine rule?

To find a side: need two sides and the angle between them

Formula: $a^2 = b^2 + c^2 - 2bc\cos A$

To find an angle: need all 3 sides

Formula: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

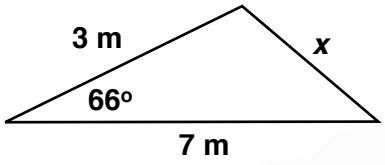


N.B. "$-a^2$" comes from the side opposite the angle we are trying to find.

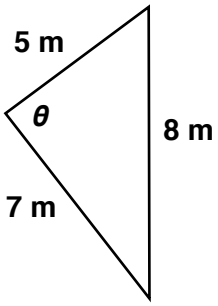
N.B. Always draw a diagram.

E.g. 2 Find the marked side or angle:

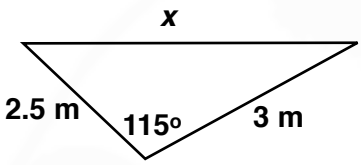
(a)



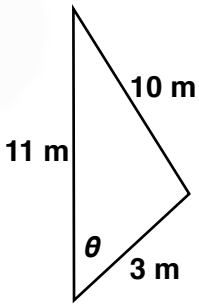
(b)



(c)



(d)



Working: (a) Finding side:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$x^2 = 3^2 + 7^2 - 2\times 7\times 3\cos 66$$
$$\therefore x = 6.40 \text{ m}$$

(b) Finding an angle:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos\theta = \frac{5^2 + 7^2 - 8^2}{2\times 5\times 7}$$
$$\theta = \cos^{-1}\left(\frac{5^2 + 7^2 - 8^2}{2\times 5\times 7}\right) = 81.8^\circ$$

E.g. 3 Find the largest angle in the triangle whose sides measure 12, 15 and 19 cm.
Hint: Which side is opposite the largest angle?

E.g. 4 Two ramblers set off walking from point P. The first rambler walks 2 km on a bearing of 025° to point A. The second rambler walks for 3 km on a bearing of 108° to point B. Find the direct distance between A and B correct to 3 s.f.

Video: Cosine rule (missing sides)
Video: Cosine rule (missing angles)

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p581 E18.2 Qu 1ace, 2-9
A*-G class textbook:	p541 E18.2 Qu 1ace, 2-9
9-1 homework book:	p196 E18.2 Qu 1-7
A*-G homework book:	p150 E18.2 Qu 1-7

Summary

To find a side: need two sides and the angle between them

Formula: $a^2 = b^2 + c^2 - 2bc\cos A$

To find an angle: need all 3 sides

Formula: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

N.B. "$-a^2$" comes from the side opposite the angle we are trying to find.

Homework book answers (only available during a lockdown)