Equation of a Tangent to a Circle

Starter

Revision of straight line graphs (Y9 material)

Gradient of line passing through (x_1, y_1) and (x_2, y_2): Gradient $= \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Equation of line with gradient m and passing through (x_1, y_1): $y - y_1 = m(x - x_1)$

Perpendicular gradients: the gradient perpendicular to $\dfrac{a}{b}$ is $-\dfrac{b}{a}$ ***negative reciprocal***

Distance between two points (x_1, y_1) and (x_2, y_2): Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

1. Find the gradient between the points $(-1, 3)$ and $(5, 7)$.

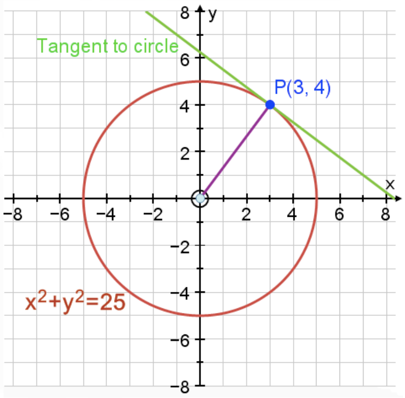
2. Find the gradient of a line which is perpendicular to a line with gradient:

(a) 6 (b) $-\dfrac{4}{5}$ (c) 2.4

3. Find the equation of the line with gradient 3 and passing through $(4, \ -1)$.

Notes

A ***tangent*** to a curve is a line that ***just touches the curve*** but does not intersect it.



The diagram shows the ***tangent*** to the circle $x^2 + y^2 = 25$ at the point $P(3, 4)$.

How could we find the ***equation of the tangent***?

If we had a point on the line and the gradient of the line we could use $y - y_1 = m(x - x_1)$ to find the equation of a straight line.

We have the point $P(3, 4)$ so what we need is the gradient of the tangent.

The tangent is perpendicular to its corresponding radius. To find the gradient of the radius, we find the gradient between the origin $(0, 0)$ and $P(3, 4)$.

Gradient of radius $= \dfrac{4 - 0}{3 - 0} = \dfrac{4}{3}$

So gradient of tangent is $-\dfrac{3}{4}$ ***negative reciprocal***

Now substitute $P(3, 4)$ and $-\dfrac{3}{4}$ into $y - y_1 = m(x - x_1)$: $y - 4 = -\dfrac{3}{4}(x - 3)$

Multiply through by 4:

$$4y - 16 = -3(x - 3)$$
$$4y - 16 = -3x + 9$$
$$3x + 4y = 25$$

Success criteria — finding the equation of a tangent to a circle

1. Find the gradient of the radius
2. Find the gradient of the tangent by doing the negative reciprocal of the gradient of the radius.
3. Find the equation of the tangent using $y - y_1 = m(x - x_1)$ or using the the $y = mx + c$ method.

E.g. 1 Find the equation of the tangent to the circles at the given point. Give your answers in the form $ax + by = k$:

(a) $x^2 + y^2 = 10$ and $(3, 1)$ (b) $x^2 + y^2 = 20$ and $(4, -2)$

(c) $x^2 + y^2 = 52$ and $(4, -6)$ (d) $x^2 + y^2 = 106$ and $(-5, -9)$

Working: (a) Gradient of radius $= \frac{1-0}{3-0} = \frac{1}{3}$

So gradient of tangent is $-\frac{3}{1} = -3$ ***negative reciprocal***

Substitute into $y - y_1 = m(x - x_1)$: $y - 1 = -3(x - 3)$

$y - 1 = -3x + 9$

Equation of tangent is $3x + y = 10$

In general:

The equation of the tangent to the circle $x^2 + y^2 = r^2$ at the point (a, b) is $ax + by = r^2$. You would not get full marks though for simply writing down the answer without showing your working.

Video: Equation of a tangent to a circle

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p407 E12.10 Qu 1-5, 7, 8, 9*
A*-G class textbook:	No exercise available
9-1 homework book:	p407 E12.10 Qu 1-7, 8*
A*-G homework book:	No exercise available

Summary

Finding the equation of a tangent to a circle:

1. Find the gradient of the radius
2. Find the gradient of the tangent by doing the negative reciprocal of the gradient of the radius.
3. Find the equation of the tangent using $y - y_1 = m(x - x_1)$ or using the the $y = mx + c$ method.

Homework book answers (only available during a lockdown)