

## Equations involving Algebraic Fractions

### Starter

1. (Review of last lesson)

Express as a single simplified fraction:

(a)  $\frac{y+3}{y+1} - \frac{y+2}{y+3}$       (b)  $\frac{x+4}{x-2} + \frac{4x-7}{(x-2)^2}$

2. (Review of previous material)

Solve:      (a)  $x^2 - 2x - 8 = 0$  by factorising  
              (b)  $3x^2 + 8x - 2 = 0$ , giving your answer to 3 s.f.

### Notes

There are several kinds of equations involving algebraic fractions.

#### **Algebraic fraction equations leading to linear equations**

1. Cross-multiply
2. Expand the brackets
3. Collect like terms

**E.g. 1** Solve: (a)  $\frac{x-2}{2} = \frac{6-x}{6}$       (b)  $\frac{x+4}{2} = \frac{x+10}{3}$       (c)  $\frac{x+2}{2} = \frac{x+4}{6}$

**Working:** (a) Cross multiply  $\frac{x-2}{2} = \frac{6-x}{6}$  to get  $6(x-2) = 2(6-x)$   
Expand:  $6x - 12 = 12 - 2x$   
 $8x = 24$   
 $x = 3$

#### **Algebraic fraction equations leading to quadratic equations**

1. Add/subtract the algebraic fractions
2. Multiply the equation by the denominator
3. Expand the brackets
4. Collect like terms and make sure the equation equal zero
5. Solve the quadratic

**E.g. 2** (a) Show that  $\frac{3}{2x-1} - \frac{4}{3x-1} = 1$  simplifies to  $x^2 - x = 0$ .  
(b) Hence solve the equation.

**Working:** (a)  $\frac{3}{2x-1} - \frac{4}{3x-1} = 1$

$$\frac{3(3x-1)}{(2x-1)(3x-1)} - \frac{4(2x-1)}{(2x-1)(3x-1)} = 1$$

$$\frac{9x-3}{(2x-1)(3x-1)} - \frac{8x-4}{(2x-1)(3x-1)} = 1$$
$$\frac{x+1}{(2x-1)(3x-1)} = 1$$

**Multiply by  $(2x-1)(3x-1)$ :**  $x+1 = (2x-1)(3x-1)$

**Expand:**  $x+1 = 6x^2 - 2x - 3x + 1$

**Collect like terms:**  $0 = 6x^2 - 6x$

**Divide by 6:**  $x^2 - x = 0$

(b)  $x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0$  or  $x = 1$

**E.g. 3** Solve these equations giving your answers to 3 s.f.:

(a)  $\frac{1}{x+1} + \frac{1}{x+2} = 1$

(b)  $\frac{2}{x+1} + \frac{5}{x+2} = 3$

**Video:** [Algebraic fractions equations](#)

[Solutions to Starter and E.g.s](#)

### Exercise

- 9-1 class textbook: p522 E16.7 Qu 1, 2-19 even  
A\*-G class textbook: p481 E16.4 Qu 1, 2-18 even  
9-1 homework book: p177 E16.7 Qu 1-7  
A\*-G homework book: p134 E16.4 Qu 1-6

### Summary

Algebraic fraction equations leading to linear equations

1. Cross-multiply
2. Expand the brackets
3. Collect like terms

Algebraic fraction equations leading to quadratic equations

1. Add/subtract the algebraic fractions
2. Multiply the equation by the denominator
3. Expand the brackets
4. Collect like terms and make sure the equation equal zero
5. Solve the quadratic

**[Homework book answers \(only available during a lockdown\)](#)**