Finding Turning Points

Starter

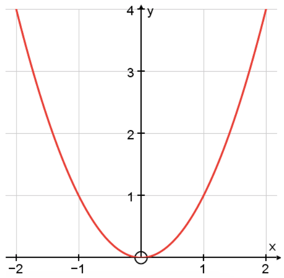
1. **(Review of last lesson)** Solve $x^2 - 2x - 7 = 0$ by completing the square, giving your answer exactly.

2. **(Review of last lesson)** Express $2x^2 - 16x - 7$ in completed square form.

Notes

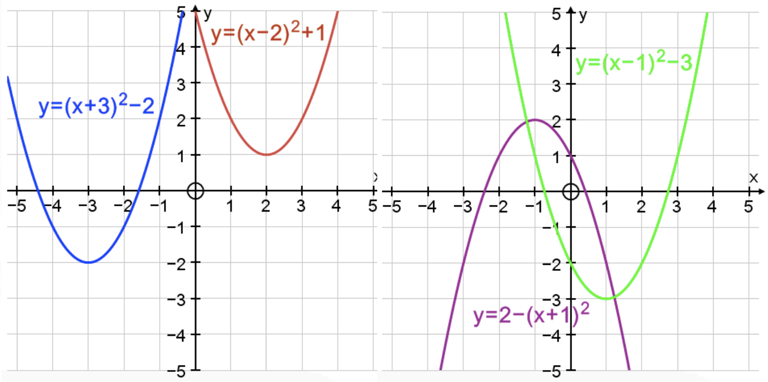
The ***vertex***, or ***turning point***, of the curve $y = x^2$ is at the point (0, 0).

The family of quadratic curves is called ***parabolas***.



E.g. 1 (a) By looking at the graphs below write down the coordinates of the vertex of the graph.

Completed square form	**Vertex**
$y = (x+3)^2 - 2$	
$y = (x-2)^2 + 1$	
$y = 2 - (x+1)^2$	
$y = (x-1)^2 - 3$	



(b) Hence conjecture where the coordinates of the curve $y = (x+p)^2 + q$ will be.

When a parabola is in completed square form $y = a(x+p)^2 + q$, we can simply write down the coordinates of the turning point:

$$(-p,\ q)$$

N.B. To find the $x-$coordinate: put the bracket equal to zero and solve
To find $y-$coordinate: the add/subtract number outside the bracket
The number multiplying the bracket does not affect the coordinates of the turning point

E.g. 2 Write down the coordinates of the turning point of these parabolas:

(a) $y = 7(x+1)^2 + 2$ (b) $y = (x-2)^2 + 3$

E.g. 3 Find the turning point of the following parabolas:

(a) $y = x^2 + 20x + 500$ (b) $y = x^2 - 9x + 11$

E.g. 4 Write down a possible equation for a parabola given that the turning point is:
(a) (3, 4) (b) (−5, 1)

Working: (a) $y = k(x - 3)^2 + 4$ where k is any number not equal to zero

N.B. Any number could be placed in front of the bracket and the turning point would be the same

Line of symmetry
The line of symmetry of a quadratic curve is the vertical line passing through the turning point.
Remember: vertical lines are of the form $x =$ ”a number”
To find the line of symmetry: ***put the expression inside the brackets equal to zero and solve***

Maximum or minimum value of a quadratic
The maximum or minimum value of a quadratics is the $y-$coordinate of the turning point
We can find the maximum (or minimum) value by putting the bracket equal to zero.

Maximum value: when the ***coefficient of x^2 is negative***
Minimum value: when the ***coefficient of x^2 is positive***

E.g. For the quadratic $y = 28 - 9(x + 5)^2$, write down
(i) the line of symmetry and
(ii) the maximum or minimum value of.

Working: (i) Put the expression inside the bracket equal to zero and solve
$x + 5 = 0$
$x = -5$ is the equation of the line of symmetry

(ii) Put the bracket equal to zero: $y = 28 - 9(0)^2 = 29$
Coefficient of x^2 is **negative** so the ***maximum*** value is 29

E.g. 5 Write down (i) the line of symmetry and (ii) the maximum or minimum value of:
(a) $y = 8(x + 6)^2 - 60$ (b) $y = 15 - 3(x - 1)^2$

Video: Quadratic graphs (completing the square)
Video: Sketching quadratics

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p399 E12.5 Qu 1-5
A*-G class textbook:	No exercise available
9-1 homework book:	p136 E12.5 Qu 1-10
A*-G homework book:	No exercise available

Summary
For $y = a(x + p)^2 + q$:

Turning point (or vertex):	$(-p,\ q)$
Line of symmetry:	$x = -p$
Maximum/minimum value:	q

Homework book answers (only available during a lockdown)