

Fractional Indices when the Numerator is 1

Starter

1. (Review of Y9 material)

Simplify: (a) $3x^7 \times 5x^6$ (b) $\frac{14x^9}{2x^4}$ (c) $(2x)^3$ (d) $5x^{-2}$

2. (Review of Y9 material) State the value of: (a) 23^0 (b) $5a^0$

Notes

Laws of indices

- $a^x \times a^y = a^{x+y}$ *when multiplying, add the indices*
- $a^x \div a^y = a^{x-y}$ *when dividing, subtract the indices*
- $(a^n)^x = a^{nx}$ *when raised to a power, multiply the indices*

Zero index: $a^0 = 1$ *a number or letter to the power of zero is 1*

Negative indices: $a^{-x} = \frac{1}{a^x}$ $\frac{1}{a^{-x}} = a^x$

Fractional indices

- Power of $\frac{1}{2}$ – square root e.g. $64^{\frac{1}{2}} = \sqrt{64} = 8$
- Power of $\frac{1}{3}$ – cube root e.g. $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$
- Power of $\frac{1}{6}$ – sixth root e.g. $64^{\frac{1}{6}} = \sqrt[6]{64} = 2$
- Power of $\frac{1}{n}$ – n-th root e.g. $64^{\frac{1}{n}} = \sqrt[n]{64}$

Useful table of powers

	To the power of 2	To the power of 3	To the power of 4	To the power of 5	To the power of 6	To the power of 7	To the power of 8
2	4	8	16	32	64	128	256
3	9	27	81	243	729	2187	
4	16	64	256	1024	4096		
5	25	125	625	3125			
6	36	216	1296				
7	49	343					

E.g. 1 Without using a calculator, find the value of:

- (a) $16^{\frac{1}{4}}$ (b) $64^{\frac{1}{3}}$ (c) $125^{\frac{1}{3}}$
 (d) $243^{\frac{1}{5}}$ (e) $729^{\frac{1}{6}}$ (f) $1296^{\frac{1}{4}}$

Working: (a) $16^{\frac{1}{4}} = (2^4)^{\frac{1}{4}} = 2^{4 \times \frac{1}{4}} = 2^1 = 2$

(b) $64^{\frac{1}{3}} = (4^3)^{\frac{1}{3}} = 4^{3 \times \frac{1}{3}} = 4^1 = 4$

E.g. 2 Without using a calculator, find the value of:

(a) $27^{-\frac{1}{3}}$ (b) $256^{-\frac{1}{4}}$ (c) $\left(\frac{81}{100}\right)^{-\frac{1}{2}}$
 (d) $\left(\frac{64}{216}\right)^{-\frac{1}{3}}$ (e) $\left(\frac{81}{625}\right)^{-\frac{1}{4}}$ (f) $\left(\frac{216}{27}\right)^{-\frac{1}{3}}$

Working: (a) $27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}} = \frac{1}{(3^3)^{\frac{1}{3}}} = \frac{1}{3^{3 \times \frac{1}{3}}} = \frac{1}{3^1} = \frac{1}{3}$
 or $27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$

(b) $256^{-\frac{1}{4}} = \frac{1}{256^{\frac{1}{4}}} = \frac{1}{\sqrt[4]{256}} = \frac{1}{4}$

E.g. 3 Simplify these expressions into the form x^n where n is an integer:

(a) $\sqrt[7]{x^{14}}$ (b) $\sqrt[4]{x^5 \times x^7}$ (c) $\sqrt{\frac{x^3}{x^{15}}}$ (d) $\frac{1}{\sqrt[3]{x^4 \times x^{14}}}$

Working: (a) $\sqrt[7]{x^{14}} = (x^{14})^{\frac{1}{7}} = x^{14 \times \frac{1}{7}} = x^2$

E.g. 4 Simplify: (a) $\sqrt{16x^{10}}$ (b) $\sqrt[3]{27y^{12}}$

Working: (a) $\sqrt{16x^{10}} = \sqrt{16} \times (x^{10})^{\frac{1}{2}} = 4x^{10 \times \frac{1}{2}} = 4x^5$

Video: [Laws of indices](#)
Video: [Negative indices](#)
Video: [Fractional indices](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook: p45 E2.1 Qu 2, 3, 4, 5ace..., (numbers), 6ace... (algebra)
 A*-G class textbook: p41 E2.2 Qu 2, 3, 4, 5ace..., (numbers), 6ace... (algebra)
 9-1 homework book: p14 E2.1 Qu 1-5
 A*-G homework book: p11 E2.2 Qu 1-4

Summary

$a^x \times a^y = a^{x+y}$

when multiplying, add the indices

$a^x \div a^y = a^{x-y}$

when dividing, subtract the indices

$(a^n)^x = a^{nx}$

when raised to a power, multiply the indices

Zero index: $a^0 = 1$

a number or letter to the power of zero is 1

Negative indices: $a^{-x} = \frac{1}{a^x}$ $\frac{1}{a^{-x}} = a^x$

Fractional indices: $x^{\frac{1}{n}} = \sqrt[n]{x}$

[Homework book answers \(only available during a lockdown\)](#)