Fractional Indices when the Numerator is not 1

Starter

1. **(Review of last lesson)**

Without a calculator, evaluate: (a) $625^{-\frac{1}{4}}$ (b) $\left(\frac{243}{32}\right)^{\frac{1}{5}}$

2. **(Review of last lesson)** Simplify: (a) $\sqrt[3]{8a^{12}}$ (b) $\sqrt[4]{81t^{20}}$

Notes

So far we have looked at fractional indices such as $x^{\frac{1}{4}}$, but what does $x^{\frac{3}{4}}$ mean?

E.g. 1 **(In pairs)** Given that $\sqrt{a} = a^{\frac{1}{2}}$ and $\sqrt[3]{a} = a^{\frac{1}{3}}$, write down the following in the form a^k where k is a fraction:

(a) $\sqrt{a^5}$ (b) $\sqrt[3]{a^7}$

Working: (a) $\sqrt{a^5} = (a^5)^{\frac{1}{2}} = a^{5\times\frac{1}{2}} = a^{\frac{5}{2}}$

In general:

$$\sqrt[n]{a^x} = a^{\frac{x}{n}}$$

$$\sqrt[n]{a^x} = (a^x)^{\frac{1}{n}} \quad \text{or} \quad \sqrt[n]{a^x} = (a^{\frac{1}{n}})^x$$

$a^{\frac{x}{n}}$ ← power, ← root

N.B. When evaluating, it is easier to ***do the root first*** as you then have a smaller number to square, cube etc.

E.g. 2 Without a calculator, find the value of:

(a) $36^{\frac{3}{2}}$ (b) $27^{\frac{2}{3}}$ (c) $16^{\frac{5}{4}}$ (d) $\left(\frac{25}{49}\right)^{\frac{3}{2}}$

Working: (a) $36^{\frac{3}{2}} = \left(\sqrt{36}\right)^3 = 6^3 = 216$ ***do the root first***

With ***negative indices***, the first step is to write the reciprocal of the number and change the sign of the power.

E.g. 3 Without a calculator, evaluate:

(a) $25^{-\frac{3}{2}}$ (b) $32^{-\frac{4}{5}}$ (c) $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

N.B. Continue to do the root first.

Working: (a) $25^{-\frac{3}{2}} = \left(\frac{1}{25}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{1}{25}}\right)^{3} = \left(\frac{1}{5}\right)^{3} = \frac{1}{125}$

N.B. When a ***fraction*** is raised to a negative power, "flip" the fraction and make the power positive.

E.g. 4 Simplify: (a) $(8a^9)^{\frac{2}{3}}$ (b) $(27p^6q^3)^{\frac{4}{3}}$

(c) $(32a^5b^{10})^{\frac{4}{5}}$ (d) $\left(\frac{32a^{10}}{b^3}\right)^{-\frac{2}{5}}$

Hint: Separate the coefficients from the letters part as they need to be treated differently.

Working: (a) $(8a^9)^{\frac{2}{3}} = (\sqrt[3]{8})^2 \times a^{9\times\frac{2}{3}} = 2^2 \times a^6 = 4a^6$

Video: Laws of indices
Video: Negative indices
Video: Fractional indices

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p47 E2.2 Qu 1, 2ace…, 3-5, 6ace…
A*-G class textbook:	p43 E2.3 Qu 1, 2ace…, 3-6, 7ace…
9-1 homework book:	p14 E2.2 Qu 1-6
A*-G homework book:	p12 E2.3 Qu 1-5

Summary

$\sqrt[n]{a^x} = a^{\frac{x}{n}}$

$\sqrt[n]{a^x} = (a^x)^{\frac{1}{n}}$ or $\sqrt[n]{a^x} = (a^{\frac{1}{n}})^x$

When evaluating, it is easier to ***do the root first*** as you then have a smaller number to square, cube etc.

With ***negative indices***, the first step is to write the reciprocal of the number and change the sign of the power.

When a ***fraction*** is raised to a negative power, "flip" the fraction and make the power positive.

Homework book answers (only available during a lockdown)