

Mixed Sine and Cosine Rule

Starter

1. **(Review of last lesson)** Village B is 12 miles north of village A. Village C is 37 miles north east of village A. Find the direct distance between village B and village C, correct to 3 s.f.

Notes

The key to knowing whether to use the sine or cosine rule is drawing an arrow on your diagram.

An arrow is **only** drawn between **known angles and their opposite sides**.

Arrow \Rightarrow use the **sine rule**

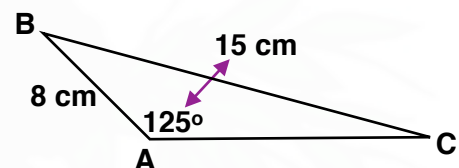
No arrow \Rightarrow use the **cosine rule**

	Side	Angle
Sine rule	Need a known angle opposite a known side $\frac{a}{\sin A} = \frac{b}{\sin B}$	Need a known angle opposite a known side $\frac{\sin A}{a} = \frac{\sin B}{b}$
Cosine rule	Need two sides and the angle between $a^2 = b^2 + c^2 - 2bc \cos A$	Need all 3 sides: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

N.B. Always draw a diagram.

E.g. 1 In a triangle ABC, $\angle BAC = 125^\circ$, BC = 15 cm and AB = 8 cm. Find $\angle ABC$.

Working: Draw a diagram.
Arrow between knowns \Rightarrow sine rule
Cannot find $\angle ABC$ directly
Need to find $\angle ACB$ first

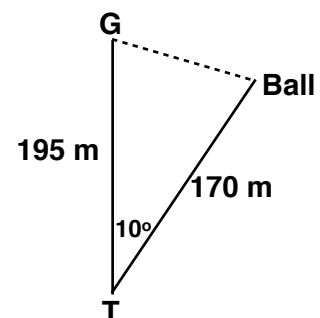


$$\begin{aligned} \text{Finding an angle: } \quad \frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin C}{8} &= \frac{\sin 125}{15} \Rightarrow \sin C = \frac{8 \sin 125}{15} \\ C &= \sin^{-1}\left(\frac{8 \sin 125}{15}\right) = 22.47^\circ \\ B &= 180 - 125 - 22.47 = 29.1^\circ \text{ (3 s.f.)} \end{aligned}$$

E.g. 2 A golfer hits her ball a distance of 170 m on a hole which measures 195 m from the tee, T, to the hole, H. If her shot is directed 10° away from the direct line to the hole, find the distance between her ball and the hole.

Working: Draw a diagram.
No arrow between knowns \Rightarrow cosine rule

$$\begin{aligned} \text{Finding side: } \quad a^2 &= b^2 + c^2 - 2bc \cos A \\ BG^2 &= 195^2 + 170^2 - 2 \times 195 \times 170 \cos 10 \\ BG &= 40.4 \\ \text{The distance between the ball and the hole} & \\ \text{is } 40.4 \text{ m (3 s.f.)} \end{aligned}$$



- E.g. 3** From a lighthouse, L, an aircraft carrier, A, is 15 km away on a bearing of 112° and a submarine, S, is 26 km away on a bearing of 200° . Find:
- the distance between A and S to 3 s.f..
 - the bearing of A from S to 1 d.p..

- E.g. 4** Starting from the same point on the ground two wires of length 25 m and 32 m are attached to different points on a vertical antenna. The angle between the wires is 11° . Find the distance between where the wires are attached to the antenna.

[Solutions to Starter and E.g.s](#)

Exercise

- 9-1 class textbook: p583 E18.3 Qu 1, 2, 3xyzxyx, 4-8; p584 E18.4 Qu 1-8
 A*-G class textbook: p543 Ex 18.3 Qu 1, 2, 3xyzxyx, 4-11; p555-557 Qu 4, 8, 9
 9-1 homework book: p197 E18.3 Qu 1-6, p198 E18.4 Qu 1-8
 A*-G homework book: p151 E18.3 Qu 1-7

Summary

An arrow is **only** drawn between **known angles and their opposite sides**.

Arrow \Rightarrow use the **sine rule**

No arrow \Rightarrow use the **cosine rule**

	Side	Angle
Sine rule	Need a known angle opposite a known side $\frac{a}{\sin A} = \frac{b}{\sin B}$	Need a known angle opposite a known side $\frac{\sin A}{a} = \frac{\sin B}{b}$
Cosine rule	Need two side and the angle between $a^2 = b^2 + c^2 - 2bc \cos A$	Need all 3 sides: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

[Homework book answers \(only available during a lockdown\)](#)