Quadratic Inequalities

Starter

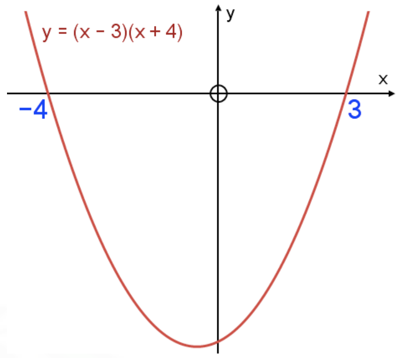
1. **(Review of last lesson)**
   Label the region R that satisfies the inequalities $x < 2$ and $x + y \geq 1$

Notes

A quadratic inequality includes a term in x^2 (e.g. $x^2 - x - 12 \leq 0$) or when brackets are expanded there will be a term in x^2 (e.g. $(x-3)(x+4) > 0$).

To solve quadratic inequalities, we need to sketch the graph.

E.g. 1 Use the sketch of the graph of $y = (x-3)(x+4)$ to solve the inequality $(x-3)(x+4) > 0$.



Sketching quadratic graphs

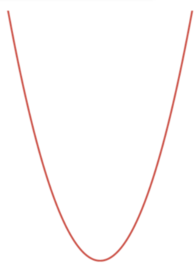
To sketch a quadratic graph of the curve $y = ax^2 + bx + c$ it is important to know the ***roots*** (i.e. where it crosses the $x-$axis) and whether the curve is ***concave-up*** or ***concave-down***.

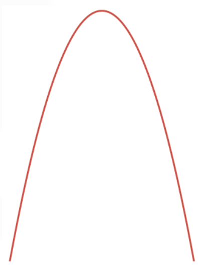
To find the roots: factorise or use the formula

N.B. If $(x-k)$ is a factor $\Leftrightarrow x = k$ is a root

Concave-up or concave-down

Coefficient of x^2 is $> 0 \Rightarrow$ concave-up **Coefficient of x^2 is $< 0 \Rightarrow$ concave-down**





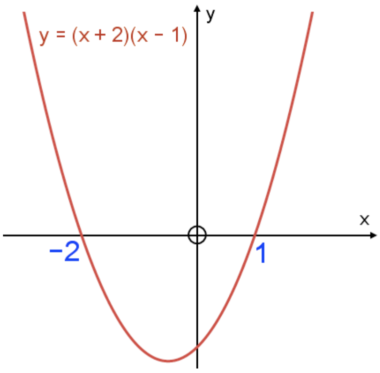
E.g. 2 Sketch the graphs of: (a) $y = (x+2)(x-1)$ (b) $y = -(x+3)(x-2)$

Working: (a) $y = (x+2)(x-1)$

$(x+2)$ is a factor $\Leftrightarrow x = -2$ is a root

$(x-1)$ is a factor $\Leftrightarrow x = 1$ is a root

Expanding the brackets the coefficient of x^2 is 1 which is > 0
$\Rightarrow$ concave-up



Success Criteria: solving quadratic inequalities

1. If necessary, rearrange so that the coefficient of x^2 is positive and the RHS is zero
2. Replace the inequality sign by an "=" sign and solve to find the roots.
3. Sketch the graph of the quadratic, using the roots as critical values and decide whether the curve is concave-up or concave-down
4. Decide whether you need the part above or below the $x-$axis
   > 0 or $\geq 0 \Rightarrow$ above the $x-$axis
   < 0 or $\leq 0 \Rightarrow$ below the $x-$axis
5. Write the solution in set notation

E.g. 3 Using your graphs from ***E.g. 2***, solve the inequalities:
(a) $(x+2)(x-1) > 0$ (b) $-(x+3)(x-2) \geq 0$
Try and give your answer in set notation.

Working: (a) > 0 so above the $x-$axis
This happens when $x < -2$ and $x > 1$
In set notation: $\{x : x < -2, x > 1\}$

E.g. 4 Solve the inequality $x^2 - x - 12 \leq 0$.
Hint: Solve $x^2 - x - 12 = 0$ to find the roots.

Video: Quadratic inequalities

Solutions to Starter and E.g.s

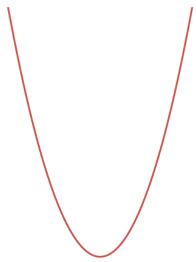
Exercise

9-1 class textbook: p515 E16.3 Qu 1-9
A*-G class textbook: No exercise
9-1 homework book: p174 E16.3 Qu 1-9
A*-G homework book: No exercise

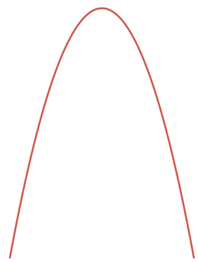
Summary

If $(x-k)$ is a factor $\Leftrightarrow x=k$ is a root

Coefficient of x^2 is $>0 \Rightarrow$ concave-up



Coefficient of x^2 is $<0 \Rightarrow$ concave-down



Solving quadratic inequalities

1. If necessary, rearrange so that the coefficient of x^2 is positive and the RHS is zero
2. Replace the inequality sign by an "=" sign and solve to find the roots.
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Homework book answers (only available during a lockdown)