Rationalise the Denominator

Starter

1. **(Review of Y11 material)** Without a calculator, evaluate $\left(\frac{25}{49}\right)^{\frac{3}{2}}$

2. **(Review of Y11 material)** Solve $25^{4x+1} = 125^2$.

Review from Year 9

N.B. When simplifying surds, look for square numbers that are factors

3. Simplify: (a) $\sqrt{8}$ (b) $\sqrt{45}$

4. Expand $(3-\sqrt{2})(3+\sqrt{2})$.

Notes

Rationalising the denominator is when the denominator of a fraction includes a surd (e.g. $\sqrt{17}$) and the aim is to ***make the denominator an integer***.

One type of rationalising was covered in Year 9.

E.g. 1 Rationalise $\frac{5}{\sqrt{17}}$.

N.B. If $\sqrt{17}$ is multiplied by $\sqrt{17}$, it becomes 17.

Working:
$$\frac{5}{\sqrt{17}} = \frac{5}{\sqrt{17}} \times \frac{\sqrt{17}}{\sqrt{17}}$$
multiplying by $\frac{\sqrt{17}}{\sqrt{17}}$ is like multiplying by 1
$$= \frac{5\sqrt{17}}{17}$$

E.g. 2 Rationalise: (a) $\frac{6}{\sqrt{10}}$ (b) $\frac{8}{3\sqrt{18}}$

More difficult rationalising

Surds like $\frac{1}{3-\sqrt{2}}$ are more difficult to rationalise.

The denominator $3-\sqrt{2}$ needs to be multiplied by a number that will get rid of the surds.

From the starter, it can be multiplied by $3+\sqrt{2}$ because $(3-\sqrt{2})(3+\sqrt{2}) = 7$

So $\dfrac{1}{3-\sqrt{2}} = \dfrac{1}{3-\sqrt{2}} \times \dfrac{3+\sqrt{2}}{3+\sqrt{2}}$ *multiplying by* $\dfrac{3+\sqrt{2}}{3+\sqrt{2}}$ *is like multiplying by 1*

$= \dfrac{3+\sqrt{2}}{7}$

N.B. Remember $\sqrt{a} \times \sqrt{a} = a$

E.g. 3 When $3+\sqrt{2}$ is multiplied by $3-\sqrt{2}$ the surd is eliminated.
Multiply the following by a suitable bracket to eliminate the surds.
Expand the two brackets to find the integer.

(a) $3+\sqrt{5}$ (b) $\sqrt{6}-2$ (c) $\sqrt{14}-\sqrt{7}$ (d) $a+\sqrt{b}$

E.g. 4 Rationalise: (a) $\dfrac{1}{\sqrt{2}-1}$ (b) $\dfrac{6}{8+\sqrt{5}}$ (c) $\dfrac{3}{\sqrt{13}-2}$

Working: (a) $\dfrac{1}{\sqrt{2}-1} = \dfrac{1}{\sqrt{2}-1} \times \dfrac{\sqrt{2}+1}{\sqrt{2}+1} = \dfrac{\sqrt{2}+1}{2-1} = \sqrt{2}+1$

E.g. 5 Rationalise: (a) $\dfrac{3}{\sqrt{3}-\sqrt{2}}$ (b) $\dfrac{4\sqrt{3}}{2\sqrt{3}-3}$ (c) $\dfrac{1}{3\sqrt{2}-2\sqrt{3}}$

Working: (a) $\dfrac{3}{\sqrt{3}-\sqrt{2}} = \dfrac{3}{\sqrt{3}-\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \dfrac{3(\sqrt{3}+\sqrt{2})}{3-2}$

$= 3(\sqrt{3}+\sqrt{2})$

E.g. 6 Rationalise: (a) $\dfrac{3-\sqrt{5}}{\sqrt{5}+1}$ (b) $\dfrac{2+\sqrt{7}}{6-\sqrt{7}}$

Working: (a) $\dfrac{3-\sqrt{5}}{\sqrt{5}+1} = \dfrac{3-\sqrt{5}}{\sqrt{5}+1} \times \dfrac{\sqrt{5}-1}{\sqrt{5}-1}$

$= \dfrac{3\sqrt{5}-3-5+\sqrt{5}}{5-1}$

$= \dfrac{4\sqrt{5}-8}{4}$

$= \sqrt{5}-2$

Video: Rationalising denominators
Video: Surds (addition)
Video: Surds (expanding brackets)

Solutions to Starter and E.g.s

Exercise
There are very questions in the textbook like the ones above.

9-1 class textbook:	p16 E1.4 Qu 5acegi, 6–9
A*-G class textbook:	p15 E1.3 Qu 7, 8acegi, 9–10
9-1 homework book:	p6 E1.4 Qu 3acegil, 4–6
A*-G homework book:	p4 E1.3 Qu 6acegil

Summary
Rationalising the denominator:

Type I: $\dfrac{a}{\sqrt{b}}$ — multiply by $\dfrac{\sqrt{b}}{\sqrt{b}}$

Type II: $\dfrac{a}{p \pm \sqrt{q}}$ — multiply by $\dfrac{p \mp \sqrt{q}}{p \mp \sqrt{q}}$ ***change the sign***

Type II: $\dfrac{a}{\sqrt{x} \pm \sqrt{y}}$ — multiply by $\dfrac{\sqrt{x} \mp \sqrt{y}}{\sqrt{x} \mp \sqrt{y}}$ ***change the sign***

Homework book answers (only available during a lockdown)