

Simultaneous Equations (Linear/Non-Linear)

Starter

1. **(Review of last lesson)** The sides of a right-angle triangle are x , $x + 1$ and $3x$. Find the value(s) of x to 3 s.f..
2. **(Review of Y9 material)** Expand $(x + 5)(x - 4)$.
3. **(Review of Y10 material)** Solve $x^2 + 3x - 10 = 0$ by factorising.

Notes

Simultaneous equations can also include a **non-linear** equation.

Linear vs. non-linear equations

Linear functions can include x and y but they are only added or subtracted

E.g. $y = 3x - 1$ $4x - 7y = 18$

Non-linear functions include x and y where at least one of them is raised to the power of 2 or more, or x and y are multiplied together.

E.g. $x^2 + 5y = 9$ $y^2 = 2x - 13$ $xy + 6x = 11$

E.g. 1 Separate these functions into linear and non-linear?

$xy = 2$ $x = y + 1$ $x(y - 1) = 3$ $x^2 + y^2 = 25$

N.B. The elimination method employed to solve simultaneous equations when both equations are linear bears no resemblance to the one we use when one of the equations is non-linear.

Success criteria – solving linear/non-linear simultaneous equations

1. Recognise the linear function and, if necessary, rearrange to make one of the unknowns the subject
2. Substitute the linear function into the non-linear function
3. Expand and rearrange to form a quadratic in x or y (remember “= 0”).
4. Solve the quadratic by either factorising or using the quadratic formula.
5. Substitute the 2 x -values (or y -values) into the rearranged linear function to find the values of y (or x).

E.g. 2 Solve the simultaneous equations $y = x^2 - 2x$ and $y = x + 4$ to find the coordinates where the curve meets the line.

Working: Linear function is $y = x + 4$ – there is no need to rearrange.
Substitute into $y = x^2 - 2x$: $x + 4 = x^2 - 2x$
 $x^2 - 3x - 4 = 0$
Factorise (or formula) to solve: Multiply: $-4 = -4 \times 1$
Add: $-3 = -4 + 1$
 $x^2 - 4x + x - 4 = 0$
 $x(x - 4) + 1(x - 4) = 0$
 $(x - 4)(x + 1) = 0$
 $x = 4$ or $x = -1$
When $x = 4$, $y = 4 + 4 = 8$
When $x = -1$, $y = -1 + 4 = 3$
The curve and line meet at the points (4, 8) and (-1, 3)

E.g. 3 Solve the equations $x + y = 5$ and $2x^2 + xy = 14$ simultaneously.

Video: [Simultaneous equations \(linear/non-linear\)](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook: p404 E12.8 Qu 7-16
A*-G class textbook: p364 E12.5 Qu 7-16
9-1 homework book: p138 E12.8 Qu 1-6
A*-G homework book: p103 E12.5 Qu 1-5

Summary

Solving linear/non-linear simultaneous equations:

1. Recognise the linear function and, if necessary, rearrange to make one of the unknowns the subject
2. Substitute the linear function into the non-linear function
3. Expand and rearrange to form a quadratic in x or y (remember “= 0”).
4. Solve the quadratic by either factorising or using the quadratic formula.
5. Substitute the 2 x -values (or y -values) into the rearranged linear function to find the values of y (or x).

[Homework book answers \(only available during a lockdown\)](#)