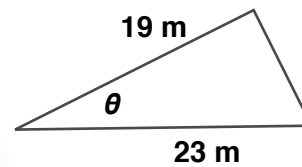
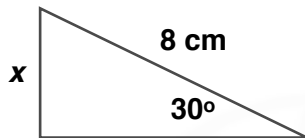


## Sine Rule

### Starter

1. Both triangles are right-angled triangles. Give your answers to 3 s.f.  
 (a) Find the value of  $x$ . (b) Find the value of  $\theta$ .  
 Remember the Japanese word for trigonometry: **SOHCAHTOA**.



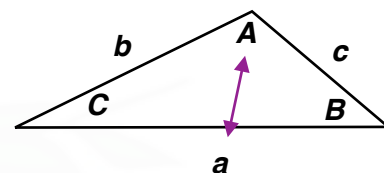
### Notes

So far we have looked at trigonometry in right-angled triangles. We will now look at trigonometry in triangles without a right-angle.

### Notation

In the triangle ABC, the following convention is used:

- **Capital letters** refer to **angles**
- **Lower case** letters are **sides**
- The side  $a$  is opposite angle  $A$



### Proof of sine rule

The area of the triangle is  $\frac{1}{2}ab \sin C$ .

We could also write the area as  $\frac{1}{2}ac \sin B$  or  $\frac{1}{2}bc \sin A$ .

$$\text{So } \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$$

Multiplying by 2 gives:  $bc \sin A = ac \sin B = ab \sin C$

Dividing by  $abc$  gives:  $\frac{bc \sin A}{abc} = \frac{ac \sin B}{abc} = \frac{ab \sin C}{abc}$

Cancelling letters:  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

In practice, we just need two parts:  $\frac{\sin A}{a} = \frac{\sin B}{b}$  — this is the **sine rule**

We can flip the fractions to get:  $\frac{a}{\sin A} = \frac{b}{\sin B}$

### Information necessary to use the sine rule

The sine rule is used when **an angle and its opposite side** are known.

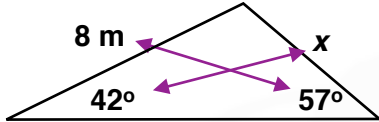
Finding angles: use  $\frac{\sin A}{a} = \frac{\sin B}{b}$

Finding sides: use  $\frac{a}{\sin A} = \frac{b}{\sin B}$

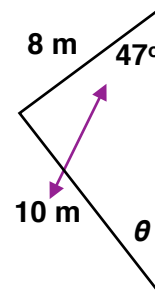
**N.B.** Always draw an arrow connecting an angle and its opposite side.  
Always draw a diagram.

**E.g. 1** Find the marked side or angle:

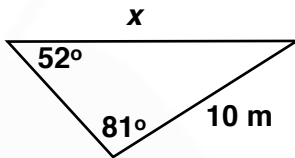
(a)



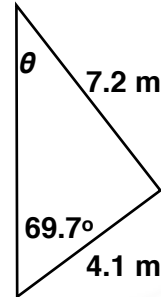
(b)



(c)



(d)



**Working:**

(a) Finding a side so use

$$\frac{x}{\sin 42} = \frac{8}{\sin 57} \Rightarrow$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$x = \frac{8 \sin 42}{\sin 57} = 6.38 \text{ m}$$

(b) Finding an angle so use

$$\frac{\sin \theta}{8} = \frac{\sin 47}{10} \Rightarrow$$

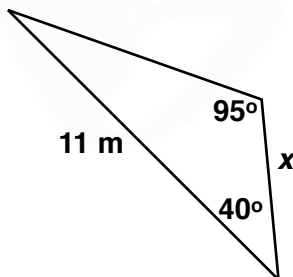
$$\theta = \sin^{-1}\left(\frac{8 \sin 47}{10}\right) = 35.8^\circ$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

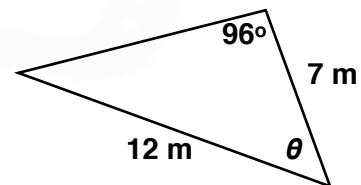
$$\sin \theta = \frac{8 \sin 47}{10}$$

**E.g. 2** Find the marked side or angle:

(a)



(b)



**E.g. 3** A triangular piece of metal PQR is such that  $\angle RPQ = 61^\circ$ , length QR = 13.1 mm and length PQ = 7.2 mm. Find the size of  $\angle PQR$ , correct to 1 dp.

Video:  
Video:

[Sine rule \(finding sides\)](#)  
[Sine rule \(finding angles\)](#)

**Exercise**

9-1 class textbook: p579 E18.1 Qu 1-6  
A\*-G class textbook: p539 E18.1 Qu 1-6  
9-1 homework book: p195 E18.1 Qu 1-9  
A\*-G homework book: p149-150 E18.1 Qu 1-8

**Summary**

Finding angles:  $\frac{\sin A}{a} = \frac{\sin B}{b}$

Finding sides:  $\frac{a}{\sin A} = \frac{b}{\sin B}$

Always draw an arrow connecting a known angle and its known opposite side.

[Homework book answers \(only available during a lockdown\)](#)