Solving Quadratics with the Formula

Starter

1. **(Review of last lesson)** Write down the turning point of the parabola $y = (x-4)^2 - 13$.

2. **(Review of last lesson)** Find the coordinates of the turning point of the parabolas:
(a) $y = x^2 + 14x + 54$. (b) $3x^2 - 36x + 2$

Notes

If we tried to solve $7x^2 - 3x - 1 = 0$ we would find it impossible to factorise and quite difficult to complete the square. The best way to solve such an equation is by using the ***quadratic formula***.

There is no need to copy the derivation of the quadratic formula.

Deriving the quadratic formula

We derive the quadratic formula by completing the square for the equation $ax^2 + bx + c = 0$ where a, b and c are constants.

$$ax^2 + bx + c = 0$$

$$a\left[x^2 + \frac{b}{a}x\right] + c = 0 \quad \textit{take the coefficient of } x^2 \textit{ out as a factor}$$

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c = 0 \quad \textit{complete the square for square brackets}$$

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] + c = 0$$

The equation can now be solved i.e. rearrange it to make x the subject.

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] = -c$$

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} = -\frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \quad \textit{same denominator}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \quad \textit{square rooting leads to } \pm$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \textit{same denominator}$$

Please start copying again.

The quadratic formula is: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Success criteria — using the quadratic formula

1. Write down the values of a, b and c
   a = coefficient of x^2 b = coefficient of x c = constant term
2. Write down the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
3. Substitute the numbers into the formula — ***put brackets around negative numbers***
4. Simplify the calculation in the square root and in the denominator
5. Split the calculation up in to the two solutions i.e. $\dfrac{p + \sqrt{q}}{r}$ and $\dfrac{p - \sqrt{q}}{r}$.
6. Use your calculator to find the two values.

E.g. Solve $7x^2 - 3x - 1 = 0$ using the quadratic formula.

Working: $a = 7$ $b = -3$ $c = -1$

Using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 7 \times (-1)}}{2 \times 7}$

$x = \dfrac{3 \pm \sqrt{9 + 28}}{14}$

$x = \dfrac{3 + \sqrt{37}}{14}$ or $x = \dfrac{3 - \sqrt{37}}{14}$

$x = 0.649$ or $x = -0.220$

Hints

- At the start, make sure the equation equals zero
- Also, make sure the equation is in the right order i.e. the x^2 term is before the x term, which is before the constant term
- When b is —ve, $-b$ becomes +ve
- When substituting —ve numbers into the formula, put them in brackets
- When squaring negative numbers, the answer is positive

E.g. 1 Solve $x^2 + x - 8 = 0$ giving your answer to 3 s.f.

Working: $a = 1$ $b = 1$ $c = -8$

Using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $x = \dfrac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-8)}}{2 \times 1}$

$x = \dfrac{1 \pm \sqrt{1 + 32}}{2}$

$x = \dfrac{1 + \sqrt{33}}{2}$ or $x = \dfrac{1 - \sqrt{33}}{2}$

$x = 2.37$ or $x = -3.37$

N.B. A question that involves solving a quadratic equation and includes the phrase "giving your answer to 3 s.f." means it will not factorise — so you should use the quadratic formula

E.g. 2 Solve these equation, giving your answer to 3 s.f.

(a) $x^2 - 2x - 5 = 0$ (b) $3x^2 + 7x - 5 = 0$

E.g. 3 Solve these equation, giving your answer to 3 s.f.

(a) $5x + x^2 - 4 = 0$ (b) $5 - 9x + 2x^2 = 0$

Hint: Rewrite the equation with terms in the correct order.

Make sure the equation equals zero before using the formula — it is best to have the coefficient of x^2 ***positive***.

E.g. 4 Solve these equations, giving your answer exactly:

(a) $x(3x + 1) = 5$ (b) $x^2 - 5x + 11 = 2x + 3$

N.B. In this case an exact answer means leaving it in surd form.

Video: Deriving the quadratic formula
Video: Quadratic formula

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p401 E12.6 Qu 1-18 odd (3 s.f.), 19-24 (p ± √q / r)
A*-G class textbook:	p360 E12.3 Qu 1-18 odd (3 s.f.), 19-24 (p ± √q / r)
9-1 homework book:	p137 E12.6 Qu 1-12
A*-G homework book:	p101 E12.3 Qu 1-12

Summary

For $ax^2 + bx + c = 0$ the quadratic formula is: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- Write out the values of a, b and c.
- When b is —ve, $-b$ becomes +ve
- When substituting —ve numbers into the formula, put them in brackets
- When squaring negative numbers, the answer is positive

Homework book answers (only available during a lockdown)