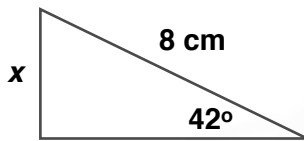


Trigonometry in 3-D

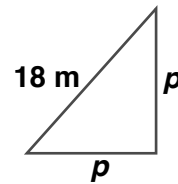
Starter

1. (Review of Y9 material) Find the missing side or angle in these right angle triangles:
N.B. We do not use the sine or cosine rule in right-angle triangles

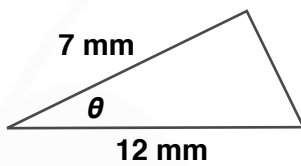
(a)



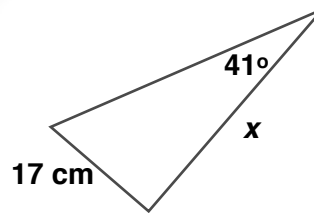
(b)



(c)



(d)



Notes

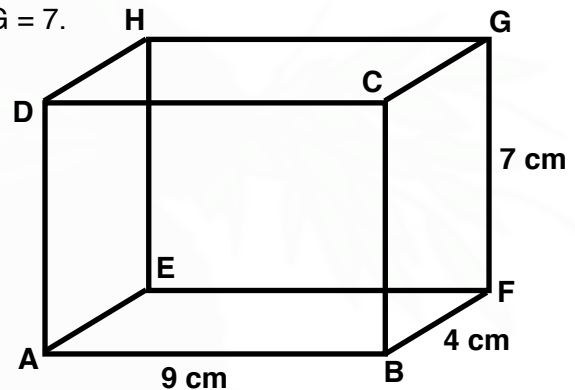
When using Pythagoras and trigonometry in 3-D, *always draw the 2-D triangle* you are working on.

N.B. Draw the line you are asked to find and draw a right-angled triangle around it.

E.g. 1 In the cuboid ABCDEFGH, AB = 9, BF = 4, FG = 7.

Find the length of:

- (a) AF
 (b) AG

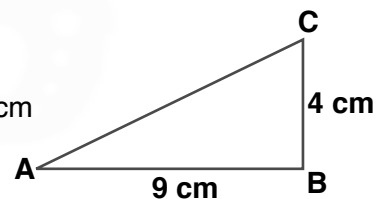


Working: (a) By Pythagoras:

$$4^2 + 9^2 = AF^2$$

$$AF^2 = 97$$

$$AF = 9.85 \text{ cm}$$



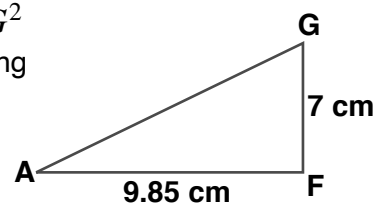
(b) By Pythagoras: $7^2 + AF^2 = AG^2$

Replace AF^2 by 97 rather than replacing AF by 9.85 to avoid rounding error.

$$7^2 + 97 = AG^2$$

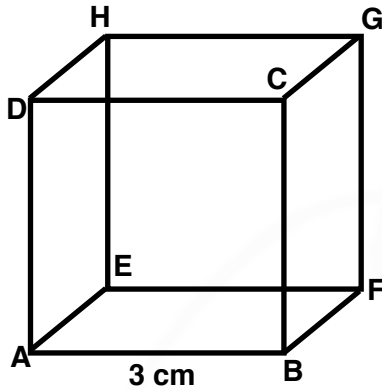
$$AG^2 = 146$$

$$AG = 12.1 \text{ cm}$$



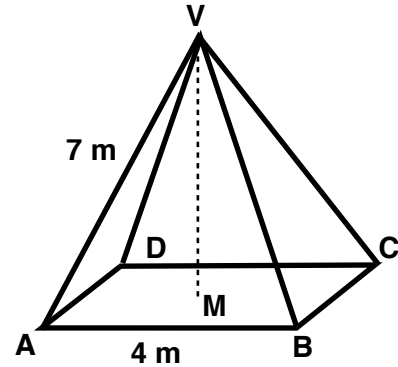
E.g. 2 A cube has sides of length 3 cm. Find:

- (a) the exact value of AF
- (b) the exact value of DF
- (c) the angle DFA.



E.g. 3 A square-based pyramid has base ABCD where AB = 4 m. The point M is the midpoint of the square, which is vertically below the vertex, V. Given that AV = 7 m, find:

- (a) the exact length of AC
- (b) the exact vertical height of the pyramid (i.e. MV)
- (c) the $\angle AVB$



Distance between 2 points in 3-D

In 2-D, to find the distance between two points (x_1, y_1) and (x_2, y_2) we use Pythagoras. The formula is:

$$\text{Distance in 2-D} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In 3-D, we can extend Pythagoras to the points (x_1, y_1, z_1) and (x_2, y_2, z_2) :

$$\text{Distance in 3-D} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

E.g. 4 Find the distance between:

- (a) (1, 4) and (13, 9)
- (b) (1, 3, 4) and (2, 8, 7)

Working: (a) Label the points: $(1, 4)$ and $(13, 9)$
 (x_1, y_1) (x_2, y_2)

$$\begin{aligned} \text{Distance in 2-D} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(13 - 1)^2 + (9 - 4)^2} \\ &= \sqrt{12^2 + 5^2} \\ &= 13 \text{ units} \end{aligned}$$

Video: [Trigonometry in 3-D](#)

[Solutions to Starter and E.g.s](#)

Exercise

- 9-1 class textbook: p587 E18.5 Qu 1-10
- A*-G class textbook: p546 E18.4 Qu 1-10
- 9-1 homework book: p199 E18.5 Qu 1-6
- A*-G homework book: p152 E18.4 Qu 1-5

Summary

When using Pythagoras and trigonometry in 3-D, *always draw the 2-D triangle*.

Distance between 2

$$\text{Distance in 2-D} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Distance in 3-D} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Homework book answers (only available during a lockdown)

