Algebraic proof

Starter

1. **(Review of last lesson)** Solve $\frac{x+3}{4} = \frac{x+9}{7}$.

Working: $\frac{x+3}{4} = \frac{x+9}{7}$

Cross multiply: $7(x+3) = 4(x+9)$
Expand: $7x + 21 = 4x + 36$
Collect like terms: $3x = 15$
Divide by 3: $x = 5$

2. **(Review of last lesson)** Solve $\frac{4}{x-2} + \frac{7}{x+1} = 3$, giving your answers to 3 s.f.

Working: $\frac{4}{x-2} + \frac{7}{x+1} = 3$

$\frac{4(x+1)}{(x-2)(x+1)} + \frac{7(x-2)}{(x-2)(x+1)} = 3$ ***common denominator***

$\frac{4x+4}{(x-2)(x+1)} + \frac{7x-14}{(x-2)(x+1)} = 3$ ***expand numerators***

$\frac{11x-10}{(x-2)(x+1)} = 3$ ***add the fractions***

Multiply by $(x+1)(x+2)$***:*** $11x - 10 = 3(x-2)(x+1)$
Expand: $11x - 10 = 3(x^2 - x - 2)$
$11x - 10 = 3x^2 - 3x - 6$
Collect like terms: $0 = 3x^2 - 14x + 4$
Use the formula: $a = 3 \quad b = -14 \quad c = 4$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4 \times 3 \times 4}}{2 \times 3}$

$x = \frac{14 \pm \sqrt{196 - -48}}{6}$

$x = \frac{14 + \sqrt{244}}{6}$ or $x = \frac{14 - \sqrt{244}}{6}$

$x = 4.36$ or $x = 0.306$ (3 s.f.)

E.g. 1 Prove that the sum of two odd numbers is even.

Working: Let the 2 odd numbers be $2x + 1$ and $2y + 1$
$2x + 1 + 2y + 1 = 2(x + y + 1)$ – which is in the form of an even number.

E.g. 2 Prove that the answer to every line of the pattern below is 8:

$3 \times 5 - 1 \times 7$ $\quad$ $4 \times 6 - 2 \times 8$ $\quad$ $5 \times 7 - 3 \times 9$

Hint: look at how the digits of each calculation are connected to each other.

Working: The nth line of pattern is $(n+2)(n+4) - n(n+6)$

Expand: $n^2 + 4n + 2n + 8 - n^2 - 6n$

Collect like terms: $n^2 + 4n + 2n + 8 - n^2 - 6n = 8$

E.g. 3 If a and b are both odd, prove that ab is odd.

Working: ***Remember*** an odd number is of the form $2n + 1$

Let a and b be odd numbers such that $a = 2x + 1$ and $b = 2y + 1$.

Then $ab = (2x+1)(2y+1)$

$= 4xy + 2x + 2y + 1$

$= 2(2xy + x + y) + 1$

which is of the form $2 \times$ a number $+ 1$

i.e. it is an odd number

Therefore, the product of 2 odd numbers is always odd

N.B. a and b cannot be $2x + 1$ and $2x + 3$ as this would be 2 ***consecutive*** odd numbers

E.g. 4 For three consecutive numbers, prove that the product of the first and third number is one less than the square of the middle number.

Working: Let the consecutive numbers be n, $n + 1$ and $n + 2$.

Product of 1st and 3rd: $n(n+2) = n^2 + 2n$

Square of middle number: $(n+1)^2 = n^2 + 2n + 1$

$(n^2 + 2n + 1) - 1 = n^2 + 2n$

So one less than the square of the middle number equals the product of the 1st and 3rd number.

Video: Algebraic proof

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p525 E16.8 Qu 1-18
A*-G class textbook:	p483 E16.5 Qu 1-14
9-1 homework book:	p178 E16.8 Qu 1-10
A*-G homework book:	p134 E16.5 Qu 1-8

Homework book answers (only available during a lockdown)