Angle between a Line and a Plane

Starter

1. **(Review of last lesson)** Consider the isosceles triangular prism in the diagram. M is the midpoint of AC and is vertically below B.

$\angle ACB = 59^o$

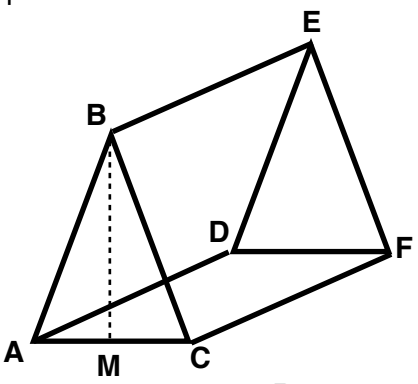
AB = 10 m

CF = 25 m.

Find

(a) the perpendicular height BM to 3 s.f.

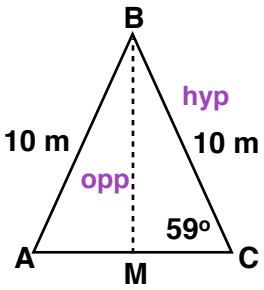
(b) the length EM to 3 s.f.



Working: (a) **Split $\triangle ABC$ into 2 equal right-angled triangles.**
Label the give side and required sides.
Choose which trig. ratio to use: sin

$$\sin 59 = \frac{BM}{10}$$

$$BM = 10 \sin 59 = 8.57 \text{ m (3 s.f.)}$$



(b) Let P be the mid-point of DF.

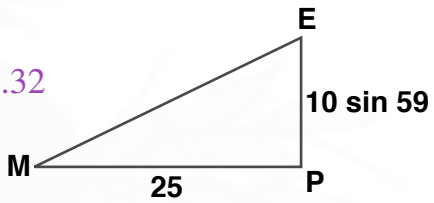
$EP = BM$ and $MP = CF$

For EP use $10 \sin 59$ instead of 8.32 to avoid rounding error.

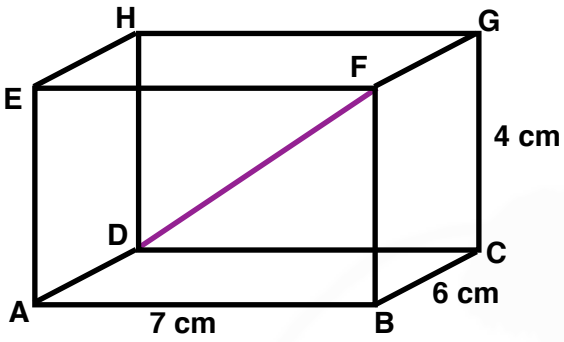
By Pythagoras,

$$EM^2 = (10 \sin 59)^2 + 25^2$$

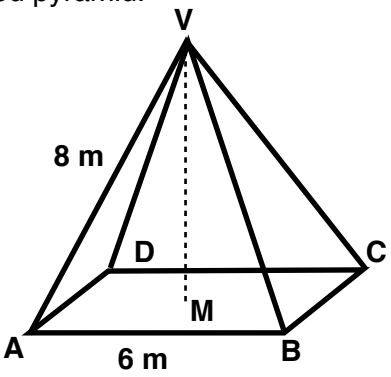
$$EM = 26.4 \text{ m (3 s.f.)}$$



E.g. 1 Find the angle between the line FD and the plane ABCD.

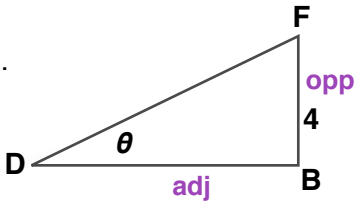


E.g. 2 Find the angle between the line AV and the plane ABCD in the square-based pyramid.

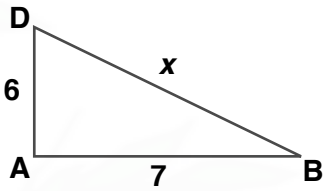


E.g. 1 Working:

Draw the right-angle triangle with FD as hypotenuse.
In $\triangle BDF$, we have one side, BF.
To find the required angle θ, we need another side.
The easiest one to find is BD.



Draw the right-angle triangle with BD as hypotenuse. We can calculate $x = BD$ since we have the other sides.



By Pythagoras,
$$x^2 = 6^2 + 7^2$$
$$x^2 = 85$$
$$x = \sqrt{85}$$

N.B. $\sqrt{85} = 9.22$ but keep $\sqrt{85}$ to avoid rounding error

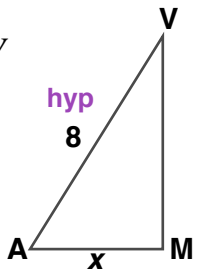
Avoid writing the decimal as it will introduce rounding error in the next step.
Label the given sides
Choose which trig. ratio to use: tan

$$\tan\theta = \frac{4}{\sqrt{85}}$$

$$\theta = \tan^{-1}\left(\frac{4}{\sqrt{85}}\right) = 23.454^{\circ}$$

The angle between the line FD and the plane ABCD is 23.5° (3 s.f.)

E.g. 2 Working: Draw the right-angle triangle with AV as hypotenuse.
To find the required angle θ, another side of the $\triangle AMV$ is needed.



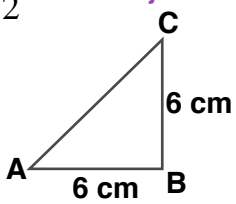
The easiest one to find is AM, which is half of AC.

By Pythagoras, $AC^2 = 6^2 + 6^2$

$AC^2 = 72$

$AC = \sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$

So $x = AM = \frac{1}{2}AC = \frac{1}{2} \times 6\sqrt{2} = 3\sqrt{2}$



Avoid writing the decimal as it will introduce rounding error in the next step.
Label the given sides
Choose which trig. ratio to use: cos

$$\cos\theta = \frac{3\sqrt{2}}{8}$$

$$\theta = \cos^{-1}\left(\frac{3\sqrt{2}}{8}\right) = 57.97^{\circ}$$

The angle between the line AV and the plane ABCD is 58.0° (3 s.f.)

Video: Angle between a line and a plane

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p591 E18.6 Qu 1-10
A*-G class textbook:	p550 E18.5 Qu 1-7
9-1 homework book:	p200 E18.6 Qu 1-6
A*-G homework book:	p153 E18.5 Qu 1-4

Homework book answers (only available during a lockdown)