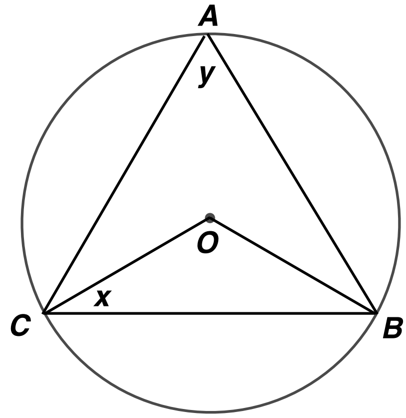
Circle Theorems Proof

Starter

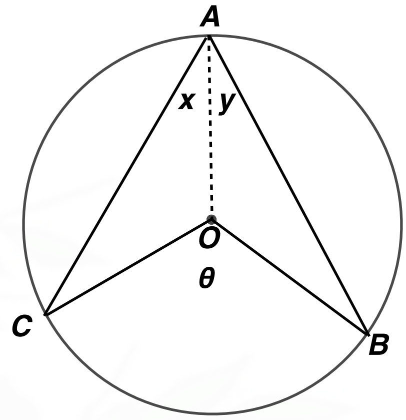
1. O is the centre of the circle. Prove that $x + y = 90^o$.



Working: $\angle COB = 2y$ because because the angle at the centre is twice the angle at the circumference from the same chord.

Since $\triangle COB$ is isosceles, $\angle OBC = x$

Angles in a triangle add up to 180^o so
$x + 2y + x = 180^o$
$\therefore 2x + 2y = 180^o$
Dividing by 2 gives $x + y = 90^o$



E.g. 1 Use the diagram to prove that the angle at the centre is twice the angle at the circumference from the same chord.
Let $\angle CAO = x$ and $\angle BAO = y$.

Hint: We need to prove that $\theta = \angle COB = 2x + 2y$

Working: Since $\triangle ACO$ is isosceles, $\angle ACO = x$.
So $\angle AOC = 180 - 2x$

Similarly, since $\triangle ABO$ is isosceles, $\angle ABO = y$.
So $\angle AOB = 180 - 2y$

There are 360^o in a full circle so $\angle COB + \angle AOC + \angle AOB = 360^o$
Substituting gives $\angle COB + 180 - 2x + 180 - 2y = 360^o$
Rearranging gives
$$\angle COB = 2x + 2y = 2(x + y) = 2\angle CAB$$
i.e. the angle at the centre is twice the angle at the circumference from the same chord.

Video: Angle in a semi circle proof
Video: Angles at the centre and circumference proof
Video: Angles at the circumference proof
Video: Cyclic quadrilateral proof
Video: Angle between radius and tangent proof
Video: Alternate segment theorem proof

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p85 E3.5 Qu 1-7, 8*
A*-G class textbook:	p78 E3.5 Qu 1-7, 8*
9-1 homework book:	p85 E3.5 Qu 1-5
A*-G homework book:	p18 E3.5 Qu 1-4

Homework book answers (only available during a lockdown)