

Completing the Square ($a = 1$)

Starter

1. **(Review of Y10 material)** Solve $x^2 + 7x + 12 = 0$ by factorising.

Working: $1 \times 12 = 12$ *coefficient of x^2 \times constant term*
Looking for 2 numbers that multiply to give 12 and add to give 7
 Multiply: $12 = 3 \times 4$
 Add: $7 = 3 + 4$
 $x^2 + 3x + 4x + 12 = 0$ *split the $7x$ term into $3x + 4x$*
 $x(x + 3) + 4(x + 3) = 0$ *factorise by grouping*
 $(x + 3)(x + 4) = 0$
 So $x = -3, x = -4$

2. Solve the equations, giving your answers to 3 s.f.:

(a) $(x + 3)^2 - 5 = 0$ (b) $(x - 7)^2 - 6 = 0$ (c) $(x - 2)^2 + 11 = 0$

Hint: Rearrange to make x the subject.

Working: (a) $(x + 3)^2 - 5 = 0$
 $(x + 3)^2 = 5$
 $x + 3 = \pm \sqrt{5}$
 $x = -3 \pm \sqrt{5}$
 $x \approx -0.764$ or $x \approx -5.24$

(b) $(x - 7)^2 - 6 = 0$
 $(x - 7)^2 = 6$
 $x - 7 = \pm \sqrt{6}$
 $x = 7 \pm \sqrt{6}$
 $x \approx 4.55$ or $x \approx 9.45$

(c) $(x - 2)^2 + 11 = 0$
 $(x - 2)^2 = -11$

No real solution since we cannot square root -11 and get a real number.

- E.g. 1** (a) Expand the bracket and collect like terms:

(i) $(x + 3)^2 - 5$

(ii) $(x + 4)^2 - 6$

- (b) What do you notice about the number in the bracket?

Working: (a) (i) $(x + 3)^2 - 5 \equiv x^2 + 6x + 9 - 5 \equiv x^2 + 6x + 4$
 (ii) $(x + 4)^2 - 6 \equiv x^2 + 8x + 16 - 6 \equiv x^2 + 8x + 10$

- (b) It is half the coefficient of x .

- E.g. 2** Express $x^2 - 6x + 11$ in completed square form.

Working:

$x^2 - 6x + 11 \equiv (x - 3)^2 \dots$

$x^2 - 6x + 11 \equiv (x - 3)^2 - (-3)^2 + 11$

$\equiv (x - 3)^2 - 9 + 11$

$\equiv (x - 3)^2 + 2$

Halve coefficient of x

Subtract square of number in bracket

We always subtract a number

Do the arithmetic

E.g. 3 Complete the square: (a) $x^2 + 2x + 2$ (b) $x^2 + 4x + 7$ (c) $x^2 - 10x + 64$

Working: (a) $x^2 + 2x + 2 \equiv (x + 1)^2 - 1^2 + 2$
 $\equiv (x + 1)^2 - 1 + 2$
 $\equiv (x + 1)^2 + 1$

(b) $x^2 + 4x + 7 \equiv (x + 2)^2 - 2^2 + 7$
 $\equiv (x + 2)^2 - 4 + 7$
 $\equiv (x + 2)^2 + 3$

(c) $x^2 - 10x + 64 \equiv (x - 5)^2 - (-5)^2 + 64$
 $\equiv (x - 5)^2 - 25 + 64$
 $\equiv (x - 5)^2 + 39$

E.g. 4 Complete the square: (a) $x^2 + 5x + 12$ (b) $x^2 - 7x - 1$

Working: (a) $x^2 + 5x + 12 \equiv (x + 2.5)^2 - 2.5^2 + 12$
 $\equiv (x + 2.5)^2 - 6.25 + 12$
 $\equiv (x + 2.5)^2 + 5.75$

(b) $x^2 - 7x - 1 \equiv (x - 3.5)^2 - 3.5^2 - 1$
 $\equiv (x - 3.5)^2 - 12.25 - 1$
 $\equiv (x - 3.5)^2 - 13.25$

Video: [Completing the square](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook: p398 E12.4 Qu 1-9
A*-G class textbook: p358 E12.2 Qu 1-9
9-1 homework book: p135 E12.4 Qu 1-3
A*-G homework book: p100 E12.2 Qu 1-3

[Homework book answers \(only available during a lockdown\)](#)