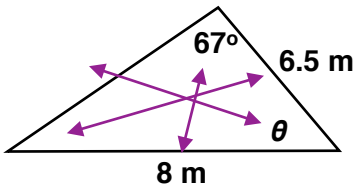
Cosine Rule

Starter

1. **(Review of last lesson)** Find the value of the missing angle to 3 s.f..



Working: The unknown angle, θ, does not have a known side opposite it.
Therefore, calculate the angle opposite 6.5.
Let the angle opposite 6.5 be x.

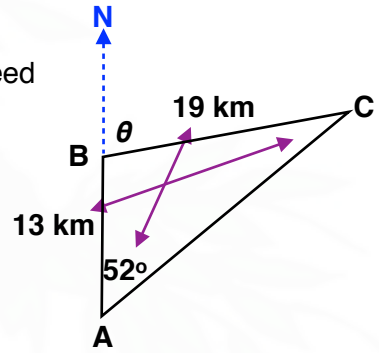
Finding an angle: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

$$\frac{\sin x}{6.5} = \frac{\sin 67}{8} \quad \Rightarrow \quad \sin x = \frac{6.5 \sin 67}{8}$$

$$x = \sin^{-1}\left(\frac{6.5 \sin 67}{8}\right) = 48.41^\circ$$

$$\theta = 180 - 67 - 48.41 = 64.6^\circ \text{ (3 s.f.)}$$

2. Point B is 13 km north of point A. Point C lies 19 km from point B and on a bearing of 052° from A. Find the bearing of C from B, to the nearest degree.



Working: The bearing of C from B is angle θ.
θ can be calculated once the angle inside the triangle at B is known — to get this, we first need the angle at C.

Finding an angle: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

$$\frac{\sin C}{13} = \frac{\sin 52}{19} \Rightarrow \sin C = \frac{13 \sin 52}{19}$$

$$x = \sin^{-1}\left(\frac{13 \sin 52}{19}\right) = 32.63^\circ$$

$$B = 180 - 52 - 32.63 = 95.37^\circ$$

$$\theta = 180 - 95.37 = 84.6^\circ \text{ (3 s.f.)}$$

The bearing of C from B is 085°.

E.g. 1 Rearrange $a^2 = b^2 + c^2 - 2bc\cos A$ to make $\cos A$ the subject.

Working:

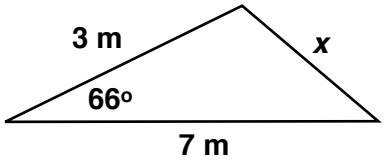
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$2bc\cos A = b^2 + c^2 - a^2 \qquad \textit{take } \cos A \textit{ to other side to make positive}$$

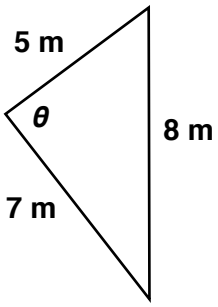
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \textit{divide by } 2bc$$

E.g. 2 Find the marked side or angle:

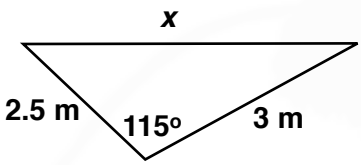
(a)



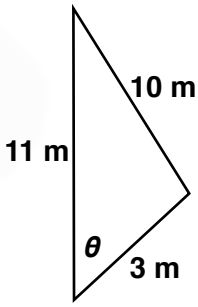
(b)



(c)



(d)



Working:

(a) Finding side: $a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 3^2 + 7^2 - 2 \times 7 \times 3\cos 66$

$\therefore x = 6.40 \text{ m}$

(b) Finding an angle: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos\theta = \dfrac{5^2 + 7^2 - 8^2}{2 \times 5 \times 7}$

$\theta = \cos^{-1}\left(\dfrac{5^2 + 7^2 - 8^2}{2 \times 5 \times 7}\right) = 81.8°$

(c) Finding side: $a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 3^2 + 2.5^2 - 2 \times 3 \times 2.5\cos 115$

$\therefore x = 4.65 \text{ m}$

(d) Finding an angle: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos\theta = \dfrac{3^2 + 11^2 - 10^2}{2 \times 3 \times 11}$

$\theta = \cos^{-1}\left(\dfrac{3^2 + 11^2 - 10^2}{2 \times 3 \times 11}\right) = 63.0°$

E.g. 3 Find the largest angle in the triangle whose sides measure 12, 15 and 19 cm.

Hint: Which side is opposite the largest angle?

Working: **The largest side is opposite the largest angle.**

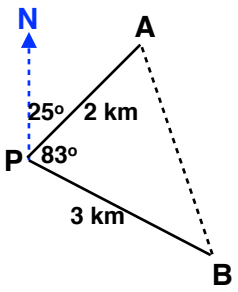
Finding an angle: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos\theta = \dfrac{12^2 + 15^2 - 19^2}{2 \times 12 \times 15}$

$\theta = \cos^{-1}\left(\dfrac{12^2 + 15^2 - 19^2}{2 \times 12 \times 15}\right) = 88.7°$

E.g. 4 Two ramblers set off walking from point P. The first rambler walks 2 km on a bearing of 025° to point A. The second rambler walks for 3 km on a bearing of 108° to point B. Find the direct distance between A and B correct to 3 s.f.

Working: **Draw a diagram.**



Angle between bearings $= 108 - 25 = 83°$

Finding side: $a^2 = b^2 + c^2 - 2bc\cos A$

$AB^2 = 2^2 + 3^2 - 2 \times 2 \times 3\cos 83$

$AB = 3.40$ km

The direct distance between A and B is 3.40 km (3 s.f.)

Video: Cosine rule (missing sides)
Video: Cosine rule (missing angles)

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p581 E18.2 Qu 1ace, 2-9
A*-G class textbook:	p541 E18.2 Qu 1ace, 2-9
9-1 homework book:	p196 E18.2 Qu 1-7
A*-G homework book:	p150 E18.2 Qu 1-7

Homework book answers (only available during a lockdown)