Defining a Region using an Inequality

Starter

1. Draw the line $x = 2$. Hence label the region R described by the inequality $x \leq 2$.

   Working: See below

2. Draw the line $y = -1$. Hence label the region R described by the inequality $y > -1$.

   Working: See below

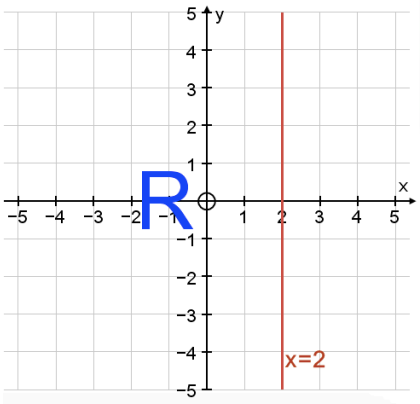
3. How will we be able to distinguish between $<$ or $>$ and $\leq$ or $\geq$?

   Working: Solid line: $\leq$ or $\geq$
   Dotted lines: $<$ or $>$

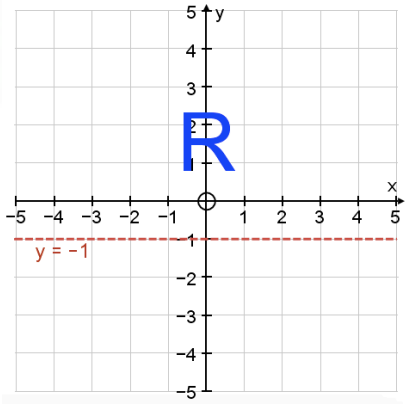
4. Draw the lines $x = -3$ and $x = 1$. Hence label the region R described by the inequality $-3 < x \leq 1$.

   Working: See below

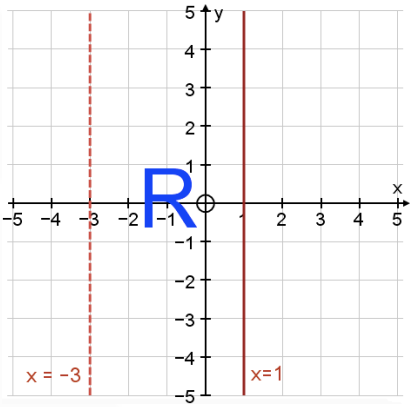
1.



2.

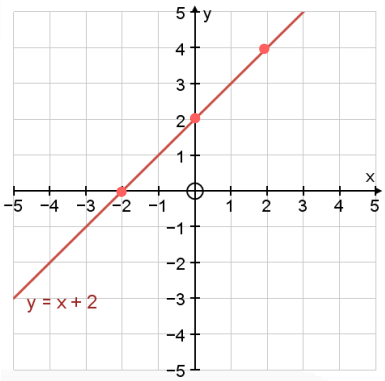


4.



E.g. 1 Draw the line $y = x + 2$.

Working: Choose $x = 0, y = 0 + 2 = 2$, plot $(0, 2)$
Choose $x = 2, y = 2 + 2 = 4$, plot $(2, 4)$
Choose a 3rd point to check:
Choose $x = -2, y = -2 + 2 = 0$,
plot $(-2, 0)$
Draw a line through the points.



N.B. The straight line $x + y = k$ passes through $(k, 0)$ and $(0, k)$. For example, the line $x + y = 5$ passes through the points $(5, 0)$ and $(0, 5)$.

Success Criteria — deciding which side of a diagonal line should be labelled R

1. Choose any point that is not on the line — usually $(0, 0)$
2. Substitute the coordinates into the inequality
3. Do the coordinates satisfy the inequality?
   Yes ⇒ the point is in required region ⇒ write R in the region where the point is
   No ⇒ the point is not in required region ⇒ write R on the opposite side of the line

E.g. 2 Decide whether the given point satisfies the inequality:

(a)	$(3, 2)$	$x + y < 6$	(b)	$(1, 5)$	$y < 3x - 1$
(c)	$(-4, 3)$	$y \geq 1 - 2x$	(d)	$(2, 6)$	$x + y \leq 8$
(e)	$(0, 0)$	$2x - 5y > 7$	(f)	$(-1, -8)$	$y \leq 3x - 5$

Working:

(a) Substitute $(3, 2)$ into $x + y < 6$: $3 + 2 < 6$ True
Yes, $(3, 2)$ does satisfy the inequality.

(b) Substitute $(1, 5)$ into $y < 3x - 1$: $5 < 3 \times 1 - 1 = 2$ False
No, $(1, 5)$ does not satisfy the inequality.

(c) Substitute $(-4, 3)$ into $y \geq 1 - 2x$: $3 \geq 1 - 2 \times -4 = 9$ False
No, $(-4, 3)$ does not satisfy the inequality.

(d) Substitute $(2, 6)$ into $x + y \leq 8$: $2 + 6 \leq 8$ True
Yes, $(2, 6)$ does satisfy the inequality.

(e) Substitute $(0, 0)$ into $2x - 5y > 7$: $0 - 0 > 7$ False
No, $(0, 0)$ does not satisfy the inequality.

(f) Substitute $(-1, -8)$ into $y \leq 3x - 5$: $-8 \leq -3 - 5$ True
Yes, $(-1, -8)$ does satisfy the inequality.

N.B. It is usually easiest to choose the point $(0, 0)$ as long as it is not on the line.

E.g. 3 Using your graph from ***E.g. 1***, label the region R described by the inequality $y \geq x + 2$.

Working:

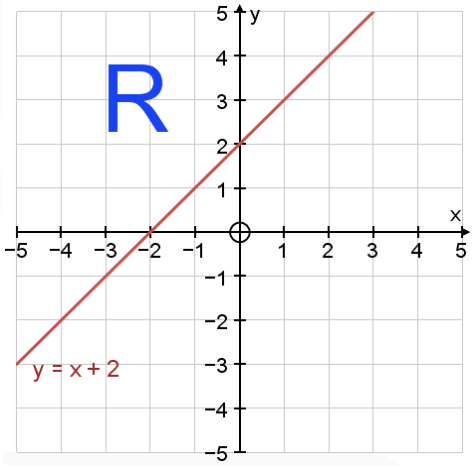
Choose the point $(0, 0)$ as it is not on the line.

Substitute into $y \geq x + 2$:
$0 \geq 0 + 2$ False

$(0, 0)$ is not in the required region

Write an R on the other side of the line.

N.B. Notice that it is a solid line because the inequality is $\geq$.

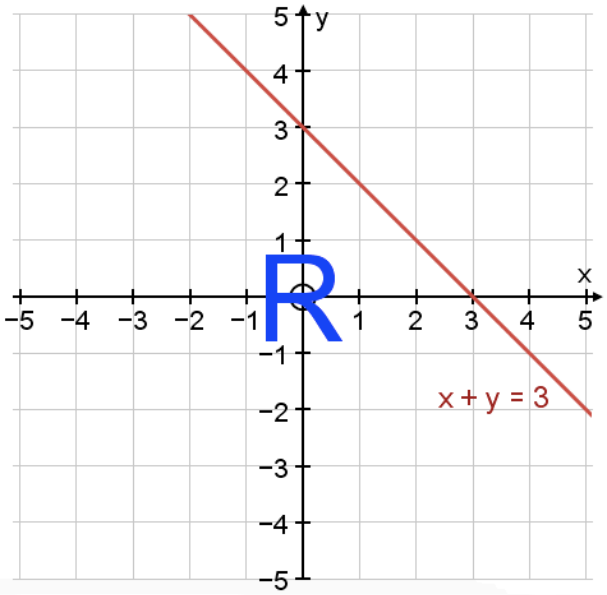


E.g. 4 Label the region R defined by the inequality:
(a) $x + y \leq 3$ (b) $y > 2x + 3$

Working: (a) Draw the line $x + y = 3$
Let $x = 0, y = 3 \Rightarrow$ plot $(0, 3)$
Let $y = 0, x = 3 \Rightarrow$ plot $(3, 0)$
To check: Let $x = 1, y = 3 - 1 = 2 \Rightarrow$ plot $(1, 2)$
Draw a solid line because the inequality is $\leq$..

Choose the point $(0, 0)$ as it is not on the line.
Substitute into $x + y \leq 3$: $0 + 0 \leq 3$ True
$(0, 0)$ is in the required region

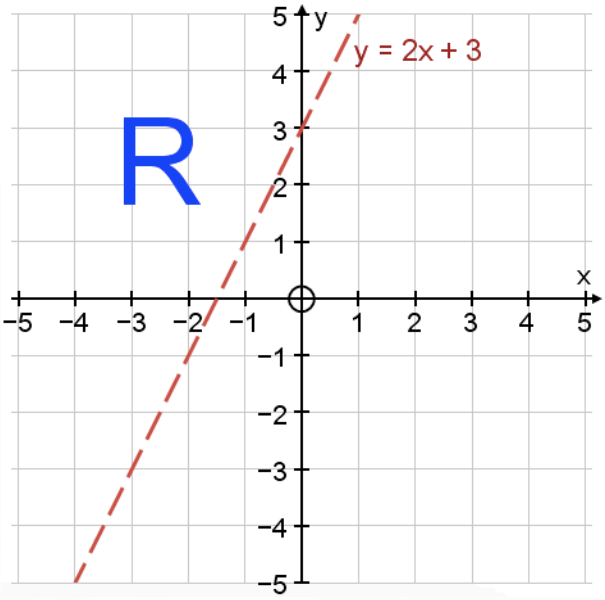
Write an R on the side of the line where $(0, 0)$ is.



(b) Draw the line $y = 2x + 3$
Let $x = 0, y = 3 \quad \Rightarrow \quad$ plot $(0, 3)$
Let $x = 1, y = 2 + 3 = 5 \quad \Rightarrow \quad$ plot $(1, 5)$
To check: Let $x = -3, y = -6 + 3 = -3 \Rightarrow$ plot $(-3, -3)$
Draw a ***dotted*** line since the inequality is $>$.

Choose the point $(0, 0)$ as it is not on the line.
Substitute into $y > 2x + 3$: $\quad 0 > 0 + 3 \quad$ False
$(0, 0)$ is not in the required region.

Write an R on the other side of the line to where $(0, 0)$ is.



Video: Regions and inequalities - horizontal and vertical lines
Video: Regions and inequalities - diagonal lines

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p512 E16.2 Qu 1, 2 (axes from −5 to 5), 3 ***Label the region R***
A*-G class textbook:	p468 M16.3 Qu 1, 2 (axes from −5 to 5), 3 ***Label the region R***
9-1 homework book:	p173 E16.2 Qu 1, 3 ***Label the region R***
A*-G homework book:	p129 M16.3 Qu 1, 3 ***Label the region R***

Homework book answers (only available during a lockdown)