Equation of a Circle

Starter

1. **(Review of last lesson)** Find the coordinates of the points where the line $y + 8 = 7x$ meets the curve $x + y = x^2 + 7$.

Working: (a) Linear function is $y + 8 = 7x$

Rearrange to $y = 7x - 8$

Replace y by $7x - 8$ in $x + y = x^2 + 7$: $x + 7x - 8 = x^2 + 7$

$$x^2 - 8x + 15 = 0$$

Solve: $15 = -5 \times -3$

$-8 = -5 + -3$

$x^2 - 5x - 3x + 15 = 0 \quad \Rightarrow \quad x(x-5) - 3(x-5) = 0$

$(x-5)(x-3) = 0 \quad \Rightarrow \quad x = 5$ or $x = 3$

When $x = 5$: $y = 7 \times 5 - 8 = 27$

When $x = 3$: $y = 7 \times 3 - 8 = 13$

(3, 13) and (5, 27)

2. **(Review of Y9 material)** Expand these brackets $(5y - 4)^2$.

Working: $(5y-4)^2 = (5y-4)(5y-4) = 25y^2 - 20y - 20y + 16 = 25y^2 - 40y + 16$

3. **(Review of Y11 material)**

Solving the equation $x^2 - 6 - 7x = 0$, giving your answer to 3 s.f.

Working: $x^2 - 6 - 7x = 0 \quad \Rightarrow \quad x^2 - 7x - 6 = 0$

Since the answer is required to 3 s.f., use the formula.

$a = 1 \qquad b = -7 \qquad c = -6$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $\quad x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 1 \times (-6)}}{2 \times 1}$

$$x = \frac{7 \pm \sqrt{49 + 24}}{2}$$

$x = \dfrac{7 + \sqrt{73}}{2}$ or $x = \dfrac{7 - \sqrt{73}}{2}$

$x = 7.77$ or $x = -0.772$ (3 s.f.)

E.g. 1 Write down the equation of the circle with:

(a) centre (0, 0) and radius 3 (b) centre (0, 0) and radius 2

Working: (a) $x^2 + y^2 = 9$

(b) $x^2 + y^2 = 4$

E.g. 2 Find the radius of the circle whose equation is:

(a) $x^2 + y^2 = 81$ (b) $x^2 + y^2 = 40$

Working: (a) Radius $= \sqrt{81} = 9$

(b) Radius $= \sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$

E.g. 3 Decide whether the following are equations of circles:

(a) $x^2 + y^2 = 8$ (b) $2x^2 + y^2 = 9$ (c) $x^2 - y^2 = 16$

Working:

(a) Yes — the expression is in the form $x^2 + y^2 = r^2$

(b) No — the coefficient in front of x^2 and y^2 must be the same

(c) No — the coefficient in front of x^2 and y^2 are different signs

E.g. 4 Decide whether the given point lies on the circumference of the circle. If the point does not lie on the circumference, state whether it is inside or outside the circle.

(a) (6, 8) and $x^2 + y^2 = 100$ (b) $(-3, 1)$ and $x^2 + y^2 = 6$

Working:

(a) Substitute (6, 8) into $x^2 + y^2 = 100$:

$$6^2 + 8^2 = 36 + 64 = 100$$

Yes, (6, 8) does lie on the circumference.

(b) Substitute $(-3, 1)$ into $x^2 + y^2 = 6$:

$$(-3)^2 + 1^2 = 9 + 1 = 10 \neq 6$$

No, $(-3, 1)$ does lie on the circumference.

Since $10 > 6$, the point $(-3, 1)$ lies outside the circle

E.g. 5 Find the coordinates of the points of intersection of the circle $x^2 + y^2 = 25$ and the line $x = 4$.

Working:

When $x = 4$: $4^2 + y^2 = 25 \Rightarrow y^2 = 9$

$\therefore y = \pm 3$

The points of intersection are (4, 3) and (4, −3)

Video: Equation of a circle

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p406 E12.9 Qu 2, (1, 3-6)
A*-G class textbook:	No exercise available
9-1 homework book:	p139 E12.9 Qu 1, 2, (3-6)
A*-G homework book:	No exercise available

Homework book answers (only available during a lockdown)