Equation of a Tangent to a Circle

Starter

Revision of straight line graphs (Y9 material)

1. Find the gradient between the points $(-1, 3)$ and $(5, 7)$.

Working: Label the points: $(-1, 3)$ and $(5, 7)$

$$(x_1, y_1) \qquad (x_2, y_2)$$

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 3}{5 - (-1)} = \frac{4}{6} = \frac{2}{3}$$

2. Find the gradient of a line which is perpendicular to a line with gradient:

(a) 6 (b) $-\frac{4}{5}$ (c) 2.4

Working: ***Find the negative reciprocals.***

(a) $-\frac{1}{6}$

(b) $\frac{5}{4}$

(c) $2.4 = \frac{24}{10} = \frac{12}{5}$ so the answer is $-\frac{5}{12}$

3. Find the equation of the line with gradient 3 and passing through $(4, -1)$.

Working: Using $y - y_1 = m(x - x_1)$: $y - (-1) = 3(x - 4)$

$$y + 1 = 3x - 12$$
$$y = 3x - 13$$

E.g. 1 Find the equation of the tangent to the circles at the given point. Give your answers in the form $ax + by = k$:

(a) $x^2 + y^2 = 10$ and $(3, 1)$ (b) $x^2 + y^2 = 20$ and $(4, -2)$

(c) $x^2 + y^2 = 52$ and $(4, -6)$ (d) $x^2 + y^2 = 106$ and $(-5, -9)$

Working: (a) Gradient of radius $= \frac{1 - 0}{3 - 0} = \frac{1}{3}$

So gradient of tangent is $-\frac{3}{1} = -3$ ***negative reciprocal***

Substitute into $y - y_1 = m(x - x_1)$: $y - 1 = -3(x - 3)$

$$y - 1 = -3x + 9$$

Equation of tangent is $3x + y = 10$

(b) Gradient of radius $= \frac{-2 - 0}{4 - 0} = -\frac{1}{2}$

So gradient of tangent is $\frac{2}{1} = 2$ ***negative reciprocal***

Substitute into $y - y_1 = m(x - x_1)$: $y - (-2) = 2(x - 4)$

$$y + 2 = 2x - 8$$

Equation of tangent is $2x - y = 10$

(c) Gradient of radius $= \frac{-6-0}{4-0} = -\frac{3}{2}$

So gradient of tangent is $\frac{2}{3}$ ***negative reciprocal***

Substitute into $y - y_1 = m(x - x_1)$: $y-(-6) = \frac{2}{3}(x-4)$

Multiple by 3: $3y + 18 = 2x - 8$

Equation of tangent is $2x - 3y = 26$

(d) Gradient of radius $= \frac{9-0}{-5-0} = \frac{9}{5}$

So gradient of tangent is $-\frac{5}{9}$ ***negative reciprocal***

Substitute into $y - y_1 = m(x - x_1)$: $y-(-9) = -\frac{5}{9}(x-(-5))$

$y + 9 = -\frac{5}{9}(x+5)$

Multiple by 9: $9y + 81 = -5x - 25$

Equation of tangent is $5x + 9y = -106$

Video: Equation of a tangent to a circle

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p407 E12.10 Qu 1-5, 7, 8, 9*
A*-G class textbook:	No exercise available
9-1 homework book:	p407 E12.10 Qu 1-7, 8*
A*-G homework book:	No exercise available

Homework book answers (only available during a lockdown)