

Equations involving Algebraic Fractions

Starter

1. (Review of last lesson)

Express as a single simplified fraction:

(a) $\frac{y+3}{y+1} - \frac{y+2}{y+3}$

(b) $\frac{x+4}{x-2} + \frac{4x-7}{(x-2)^2}$

Working:

$$\begin{aligned} \text{(a)} \quad \frac{y+3}{y+1} - \frac{y+2}{y+3} &= \frac{(y+3)(y+3)}{(y+1)(y+3)} - \frac{(y+2)(y+1)}{(y+1)(y+3)} \\ &= \frac{y^2+3y+3y+9}{(y+1)(y+3)} - \frac{y^2+y+2y+2}{(y+1)(y+3)} \\ &= \frac{y^2+6y+9}{(y+1)(y+3)} - \frac{y^2+3y+2}{(y+1)(y+3)} \\ &= \frac{y^2+6y+9 - (y^2+3y+2)}{(y+1)(y+3)} \\ &= \frac{3y-7}{(y+1)(y+3)} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{x+4}{x-2} + \frac{4x-7}{(x-2)^2} &= \frac{(x+4)(x-2)}{(x-2)^2} + \frac{4x-7}{(x-2)^2} \\ &= \frac{x^2-2x+4x-8}{(x-2)^2} + \frac{4x-7}{(x-2)^2} \\ &= \frac{x^2+2x-8}{(x-2)^2} + \frac{4x-7}{(x-2)^2} \\ &= \frac{x^2+6x-15}{(x-2)^2} \end{aligned}$$

2. (Review of previous material)

- Solve:
- (a) $x^2 - 2x - 8 = 0$ by factorising
- (b) $3x^2 + 8x - 2 = 0$, giving your answer to 3 s.f.

Working:

$$\begin{aligned} \text{(a)} \quad x^2 - 2x - 8 &= x^2 - 4x + 2x - 8 \\ &= x(x-4) - 2(x-4) \\ &= (x-4)(x-2) \end{aligned}$$

$$\therefore (x-4)(x-2) = 0$$

$$x = 4 \quad \text{or} \quad x = -2$$

(b) $a = 3 \quad b = 8 \quad c = -2$

$$\begin{aligned} \text{Using } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}: \quad x = \frac{-8 \pm \sqrt{8^2 - 4 \times 3 \times (-2)}}{2 \times 3} \\ &= \frac{-8 \pm \sqrt{64 - -24}}{6} \end{aligned}$$

$$\begin{aligned} x &= \frac{-8 + \sqrt{88}}{6} \quad \text{or} \quad x = \frac{-8 - \sqrt{88}}{6} \\ x &= 0.230 \quad \text{or} \quad x = -2.90 \quad (3 \text{ s.f.}) \end{aligned}$$

E.g. 1 Solve: (a) $\frac{x-2}{2} = \frac{6-x}{6}$ (b) $\frac{x+4}{2} = \frac{x+10}{3}$ (c) $\frac{x+2}{2} = \frac{x+4}{6}$

Working:

(a) Cross multiply $\frac{x-2}{2} = \frac{6-x}{6}$ to get $6(x-2) = 2(6-x)$
 Expand: $6x - 12 = 12 - 2x$
 $8x = 24$
 $x = 3$

(b) Cross multiply $\frac{x+4}{2} = \frac{x+10}{3}$ to get $3(x+4) = 2(x+10)$
 Expand: $3x + 12 = 2x + 20$
 $x = 8$

(c) Cross multiply $\frac{x+2}{2} = \frac{x+4}{6}$ to get $6(x+2) = 2(x+4)$
 Expand: $6x + 12 = 2x + 8$
 $4x = -4$
 $x = -1$

E.g. 2 (a) Show that $\frac{3}{2x-1} - \frac{4}{3x-1} = 1$ simplifies to $x^2 - x = 0$.
 (b) Hence solve the equation.

Working:

(a) $\frac{3}{2x-1} - \frac{4}{3x-1} = 1$

$$\frac{3(3x-1)}{(2x-1)(3x-1)} - \frac{4(2x-1)}{(2x-1)(3x-1)} = 1$$

$$\frac{9x-3}{(2x-1)(3x-1)} - \frac{8x-4}{(2x-1)(3x-1)} = 1$$

$$\frac{x+1}{(2x-1)(3x-1)} = 1$$

Multiply by $(2x-1)(3x-1)$: $x+1 = (2x-1)(3x-1)$
Expand: $x+1 = 6x^2 - 2x - 3x + 1$
Collect like terms: $0 = 6x^2 - 6x$
Divide by 6: $x^2 - x = 0$

(b) $x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0$ or $x = 1$

E.g. 3 Solve these equations giving your answers to 3 s.f.:

(a) $\frac{1}{x+1} + \frac{1}{x+2} = 1$

(b) $\frac{2}{x+1} + \frac{5}{x+2} = 3$

Working:

(a) $\frac{1}{x+1} + \frac{1}{x+2} = 1$

$$\frac{\frac{1}{x+2} + \frac{x+1}{(x+1)(x+2)}}{(x+1)(x+2)} = 1 \quad \text{common denominator}$$

$$\frac{2x+3}{(x+1)(x+2)} = 1 \quad \text{add the fractions}$$

Multiply by $(x+1)(x+2)$: $2x+3 = (x+1)(x+2)$

Expand: $2x+3 = x^2+3x+2$

Collect like terms: $0 = x^2+x-1$

Use the formula: $a = 1 \quad b = 1 \quad c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \quad x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$x = \frac{-1 \pm \sqrt{1 - -4}}{2}$$

$$x = \frac{-1 + \sqrt{5}}{2} \quad \text{or} \quad x = \frac{-1 - \sqrt{5}}{2}$$

$$x = 0.618 \quad \text{or} \quad x = -1.62 \quad (3 \text{ s.f.})$$

(b) $\frac{2}{x+1} + \frac{5}{x+2} = 3$

$$\frac{\frac{2}{x+2} + \frac{5(x+1)}{(x+1)(x+2)}}{(x+1)(x+2)} = 3 \quad \text{common denominator}$$

$$\frac{7x+9}{(x+1)(x+2)} = 3 \quad \text{expand numerators}$$

$$\frac{7x+9}{(x+1)(x+2)} = 3 \quad \text{add the fractions}$$

Multiply by $(x+1)(x+2)$: $7x+9 = 3(x+1)(x+2)$

Expand: $7x+9 = 3(x^2+3x+2)$

$7x+9 = 3x^2+9x+6$

Collect like terms: $0 = 3x^2+2x-3$

Use the formula: $a = 3 \quad b = 2 \quad c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \quad x = \frac{-2 \pm \sqrt{2^2 - 4 \times 3 \times (-3)}}{2 \times 3}$$

$$x = \frac{-2 \pm \sqrt{4 - -36}}{6}$$

$$x = \frac{-2 + \sqrt{40}}{6} \quad \text{or} \quad x = \frac{-2 - \sqrt{40}}{6}$$

$$x = 0.721 \quad \text{or} \quad x = -1.39 \quad (3 \text{ s.f.})$$

Video: [Algebraic fractions equations](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook: p522 E16.7 Qu 1, 2-19 even
A*-G class textbook: p481 E16.4 Qu 1, 2-18 even
9-1 homework book: p177 E16.7 Qu 1-7
A*-G homework book: p134 E16.4 Qu 1-6

[Homework book answers \(only available during a lockdown\)](#)

