

Estimating Powers and Roots

Starter

1. (Review of last lesson) Solve $9^{32x} = 81^{7x+1}$.

Working: The number 3 connects 9 and 81.

Replace 9 by 3^2 and 81 by 3^4 : $(3^2)^{32x} = (3^4)^{7x+1}$
 3rd law of logs: $3^{64x} = 3^{28x+4}$
 Equating powers of 3: $64x = 28x + 4$
 $36x = 4$
 $x = \frac{1}{9}$

E.g. Estimate the value of 4.3^3 .

Working: The integers either side of 4.3 are 4 and 5.

Raise these to the same power as the decimals: $4^3 = 64$ and $5^3 = 125$

So $64 < 4.3^3 < 125$

Since 4.3 is closer to 4, $4.3^3 \approx 80$

N.B. An estimate between 75 and 85 would be fine

E.g. 1 Estimate 2.8^5 .

Working: The two integers are 2 and 3.

$2^5 < 2.8^5 < 3^5$ *consider the two integers around 2.8*

$32 < 2.8^5 < 243$

Since 2.8 is closer to 3 than 2, 2.8^5 will be closer to 243 than 32

$2.8^5 \approx 170$

E.g. 2 Estimate $\sqrt[3]{150}$.

Working: Find the nearest cube numbers above and below 150

$\sqrt[3]{125} = 5$ and $\sqrt[3]{216} = 6$

$\sqrt[3]{125} < \sqrt[3]{150} < \sqrt[3]{216}$

$5 < \sqrt[3]{150} < 6$

150 is closer to 125 than 216

So $\sqrt[3]{150}$ will be closer to 5 than 6

$\sqrt[3]{150} \approx 5.3$

E.g. 3 Estimate: (a) 3.6^4 (b) $\sqrt{40}$ (c) $\sqrt{12}$ (d) $\sqrt[4]{30}$

Working: (a) The two integers are 3 and 4.

$3^4 < 3.6^4 < 4^4$

consider the two integers around 2.8

$81 < 3.6^4 < 256$

Since 3.6 is closer to 4 than 3, 3.6^4 will be closer to 256 than 81

$4.2^3 \approx 170$ (between 160 and 180 is fine)

- (b) The nearest square numbers above and below 40 are 36 and 49.
 $\sqrt{36} = 6$ and $\sqrt{49} = 7$
 $\sqrt{36} < \sqrt{40} < \sqrt{49}$
 $6 < \sqrt{40} < 7$
40 is closer to 36 than 49
So $\sqrt{40}$ will be closer to 6 than 7
 $\sqrt{40} \approx 6.3$ (between 6.2 and 6.4 would be fine)
- (c) The nearest square numbers above and below 12 are 9 and 16
 $\sqrt{9} = 3$ and $\sqrt{16} = 4$
 $\sqrt{9} < \sqrt{12} < \sqrt{16}$
 $3 < \sqrt{12} < 4$
12 is closer to 9 than 16 so $\sqrt{12}$ will be closer to 3 than 4
 $\sqrt{12} \approx 3.4$ (between 3.4 and 3.5 would be fine)
- (d) The nearest quartic numbers above and below 30 are 16 and 81
 $\sqrt[4]{16} = 2$ and $\sqrt[4]{81} = 3$
 $\sqrt[4]{16} < \sqrt[4]{30} < \sqrt[4]{81}$
 $2 < \sqrt[4]{30} < 3$
30 is closer to 16 than 81 so $\sqrt[4]{30}$ will be closer to 2 than 3
 $\sqrt[4]{30} \approx 2.3$ (between 2.2 and 2.4 would be fine)

E.g. 4 Estimate these calculations, showing your working:

(a)
$$\frac{19.7 + \sqrt{15}}{7.8}$$

(b)
$$\frac{10.7 + 2.1^3}{\sqrt[3]{120}}$$

Working: (a)
$$\frac{19.7 + \sqrt{15}}{7.8} \approx \frac{20 + 4}{8} = 3$$

(b)
$$\frac{10.7 + 2.1^3}{\sqrt[3]{120}} \approx \frac{11 + 9}{5} = 4$$

Video: <https://www.youtube.com/watch?v=GWiUL0c8eJA>

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook: p49 E2.4 Qu 3-7
A*-G class textbook: No exercise
9-1 homework book: p17 E2.4 Qu 2-10
A*-G homework book: No exercise

Homework book answers (only available during a lockdown)